

FINITE ELEMENT ANALYSIS OF
OUT-OF-PLANE DISTORTION OF
WELDED PANEL STRUCTURES

Dong Baek Shin

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BY

LCDR. DONG BAEK SHIN, ROKN

B.S., REPUBLIC OF KOREA NAVAL ACADEMY
(1960)

B.S., SEOUL NATIONAL UNIVERSITY
(1966)

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ABSTRACT

Two dimensional distortions of a panel structure due to welding are analyzed by the finite element displacement method with an assumption of elastic deformation during welding process. Construction of stiffness matrices relevant to the welding problem is demonstrated for the one and the two dimensional cases. Computer programs are presented.

Computations have been carried out using the one dimensional experimental value of unconstrained angular change along the welded edge and its equivalent constrained welding moment as an input to the computer program.

These computed results are compared to the particular experimental values. The results indicate that an analytical approach using finite element method can predict the distortion phenomena of a complicated two dimensional structure with a reasonable degree of accuracy.

Several recommendations are made concerning further investigations aimed at evaluating relevant value of angular change and equivalent constrained welding moment.

Thesis Supervisor: Koichi Masubuchi

Title: Professor of Naval Architecture

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NOMENCLATURE

a, b	Length of a finite element in x- and y-direction, respectively
C	Equivalent welding moment for one dimensional deflection
C_x	Equivalent welding moment along y-direction as in Figure 6
C_y	Equivalent welding moment along x-direction as in Figure 6
C	Coefficient matrix
\underline{D}'	Matrix defined in equation (49)
\underline{D}	Matrix defined as \underline{D}'/β
E	Young's modulus of elasticity
$\underline{F}_x, \underline{F}_y, \underline{G}, \underline{H}, \underline{q}$	Matrix defined in Appendix A
$\underline{F}_x^T, \underline{F}_y^T, \underline{G}^T, \underline{H}^T, \underline{q}^T$	Transpose of the matrix $\underline{F}_x, \underline{F}_y, \underline{G}, \underline{H}$ and \underline{q} , respectively.
I	$t^3/12, \text{ in.}^4/\text{in.}$
\underline{k}	Stiffness matrix for one rectangular finite element
$\underline{k}_{wx}, \underline{k}_{wy}$	Additional stiffness matrix due to welding defined in equation (58b) and (58c), respectively
L_x, L_y	Length of one panel structure in x- and y-directions, respectively
M, N	Number of finite element in x- and y-directions, respectively
$\underline{P}, \underline{P}_x, \underline{P}_y, \underline{P}_{\ell x}, \underline{P}_{\ell y}$	Matrix defined in equation (64), (65), (66) and (67), respectively
$\underline{Q}_x, \underline{Q}_y$	Load matrix due to welding defined in equations (58d) and (58e), respectively
t	Thickness of plate
\underline{T}	Matrix defined in Appendix A

\underline{T}^{-1}	Inverse matrix of \underline{T}
U_T	Total strain energy
U_p	Elastic strain energy
U_w	Welding energy
w	Deflection in plate
W	Weight of electrode consumed, gram/cm.
W_i, W_j	Weight for numerical integration
x, y, z	Ordinally coordinates for a finite element
X_i, Y_j	Numerical integration points in normalized coordinates
β	Constant defined as $Et^3/12(1-\nu^2)$
ν	Poisson's ratio
θ_o	Free joint angular change
θ	Constrained angular change
δ	Variational notation

I INTRODUCTION

A. Background

Welding is used extensively in the fabrication of many structures, including ships, airplanes, buildings, pressure vessels, etc., providing many advantages over the techniques such as riveting, casting, and forging. However, distortion problems are always encountered during welding. Distortion often prevents the achievement of design tolerances, reduces joint strength by mismatching, and imparts initial deflection into structural members.

A typical structural member is a panel structure, which is composed of a flat plate and longitudinal and transverse stiffeners fillet welded to the plate, as shown in Figure 1. Angular changes produced along the fillet welds cause out-of-plane distortion of the panel. The excessive out-of-plane distortion reduces the buckling strength of the panel that is subjected to compressive loading.⁽³⁾ Corrugation failures of bottom plating in some welded cargo ships are believed to be due primarily to the reduction of buckling strength of the plating with excessive initial distortion.^(15,16,17)

Welding distortion is very complicated phenomena associated with many factors:

1. Heat conductivity of weldment which controls the distribution of temperature gradient of welded metal.
2. Thermal expansion coefficient which controls the expansion and shrinkage of welded metal.

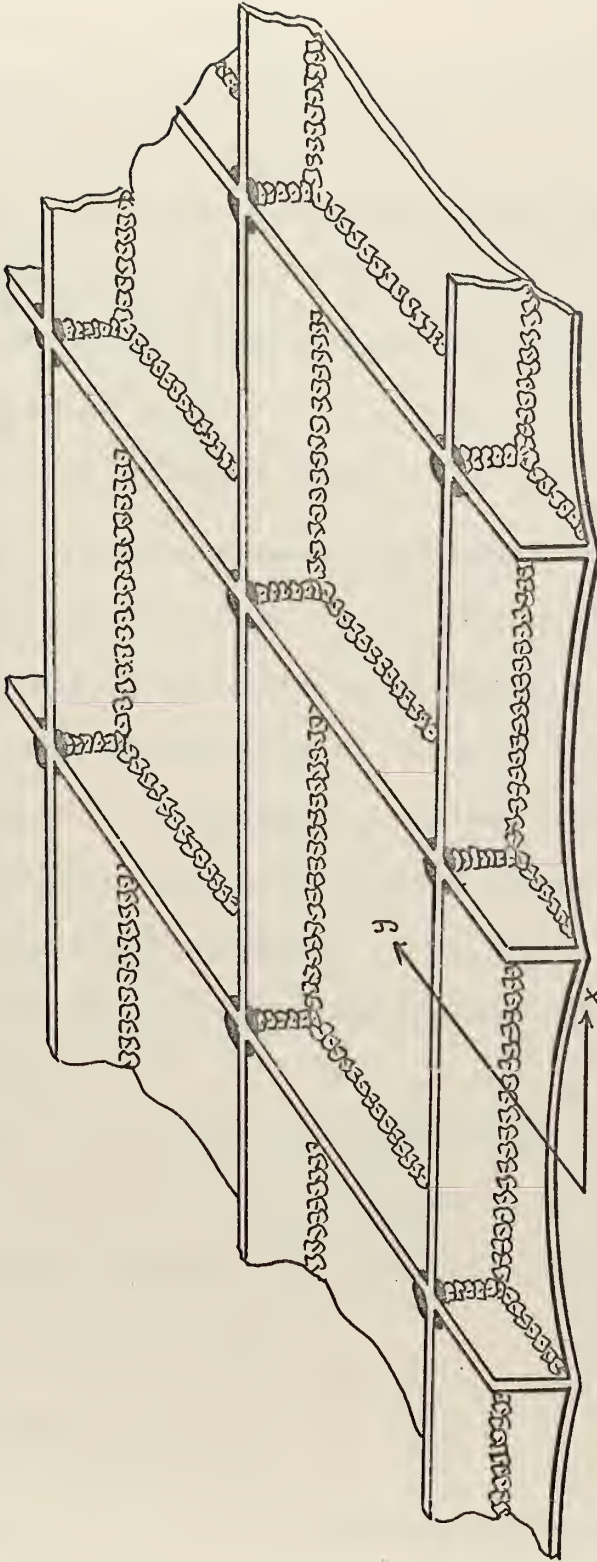


Figure 1 Panel Structure with Longitudinal and Transverse Stiffeners

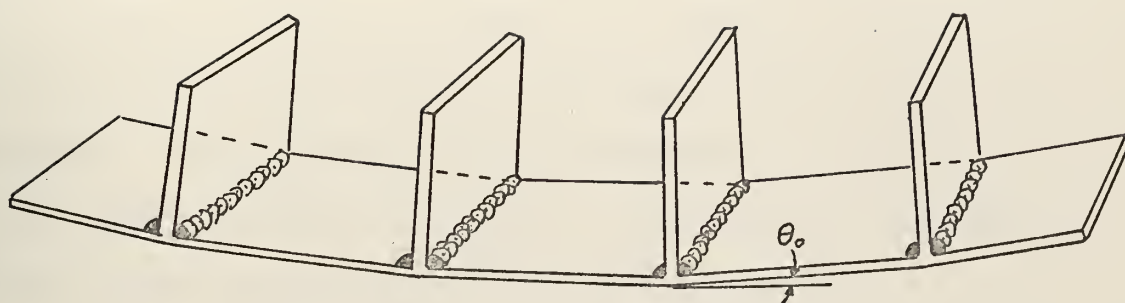
3. Moduli of elasticity which governs the rigidity against deformation.
4. Yielding strength which governs the size of the plastic zone near the welds.
5. Degree of constraint which governs the freedom of movement during weld deformation.
6. Amount of heat input used which governs the size of the melted zone.
7. Amount of weldment used.

Therefore, it is extremely difficult to analyze the welding deformation phenomena by a pure analytical way. Yet, only one dimensional deformation of panel structures has been analyzed with the assumptions of elastic deformation during welding process.⁽¹⁾ Obviously, welding deformations are neither pure elastic nor pure plastic phenomena, but the plastic zone near the weldments is so small compared to panel structural size that it is believed to be closer to the elastic behavior.

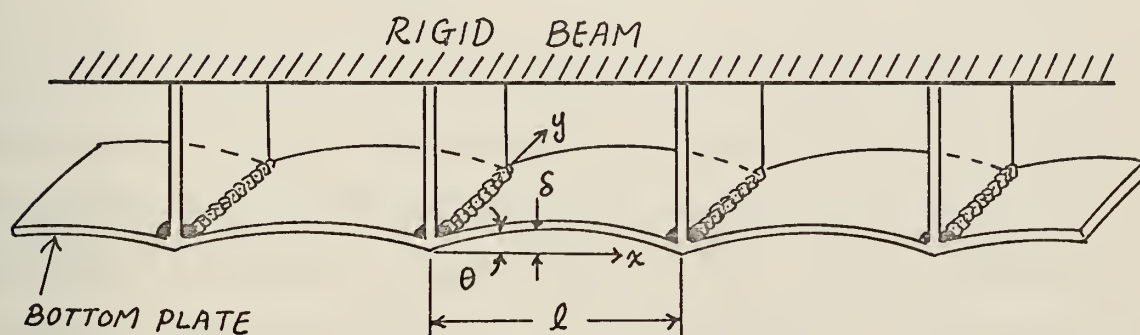
Therefore, the analytical approach of the one dimensional deformation prediction can be treated as an elastic deformation by considering moduli of elasticity, degree of constraint, and amount of weldment used.

B. Previous Investigations

Masubuchi, et. al.^(1,3) have investigated the one dimensional plate deformation during welding with a minimum strain energy concept using elastic bending theory.



a. Free Joint



b. Constrained Joint
(framed structure)

Figure 2 Distortion Caused by Angular Change in Two Types of Fillet Welded Structures

If a fillet joint is free from external constraint, the joint simply bends to a polynomial form having a knuckle at the weld as shown in Figure 2a. However, if the joint is constrained by some means, a different type of distortion is produced. For example, if the stiffeners are welded to a rigid beam, as shown in Figure 2b, the angular changes at fillet welds cause wavy or arc-form distortion.

In the simplest case in which sizes of all welds are the same, the distortions of all spans are equal and the distortion, w , which can be expressed as follows:

$$\delta/l = [1/4 - (x/l - 1/2)^2] \cdot \theta \quad (1)$$

where δ = deflection of plate

θ_o = angular change at a fillet weld with constraint

l = length of span

The amount of angular change, θ , in constrained structures is smaller than that in a free joint, θ_o . This indicates that a certain amount of energy is necessary to decrease the angular change from θ_o to θ . If the necessary energy is represented by U_w , it may be given by the following equation.

$$U_w = \int_0^{\theta_o - \theta} \frac{dU_w}{d(\theta_o - \theta)} d(\theta_o - \theta) \quad (2)$$

On the other hand, the strain energy stored in the constrained plate, U_p , can be expressed using the elastic beam theory.

$$U_p = \frac{E'I}{l} \theta^2 \quad (3)$$

where $E' = E/1 - \nu^2$

$$I = t^3/12$$

Since U_p increases but on the contrary U_w decreases as the constrained angle θ increases, the condition for equilibrium of this system requires that the total strain energy $U_t = U_w + U_p$ should be minimum. Furthermore, for the simplicity of the problem, the ratio of incremental welding energy change to angular change is assumed to be linear as in the following equation.

$$\frac{dU_w}{d(\theta_o - \theta)} = C(\theta_o - \theta) \quad (4)$$

where C = a coefficient determined by the weight of the deposited metal and by the welding procedures.

From equations (2) and (4), welding energy per unit length of width can be expressed in terms of C , θ , and θ_o ,

$$U_w = \frac{C}{2} (\theta_o - \theta)^2 \quad (5)$$

Accordingly, the condition of equilibrium is as follows:

$$\frac{\partial U_w}{\partial \theta} = C(\theta_o - \theta) + 2\frac{E'I}{\ell} = 0 \quad (6)$$

From equation (6), relation between θ and θ_o can be expressed,

$$\theta = \frac{\theta_o}{1 + \frac{2E'I}{\ell C}} \quad (7)$$

The values of C can be determined by using relationship obtained by experimentally measuring θ_o and θ . Empirical formula for the calculation of C has been proposed by

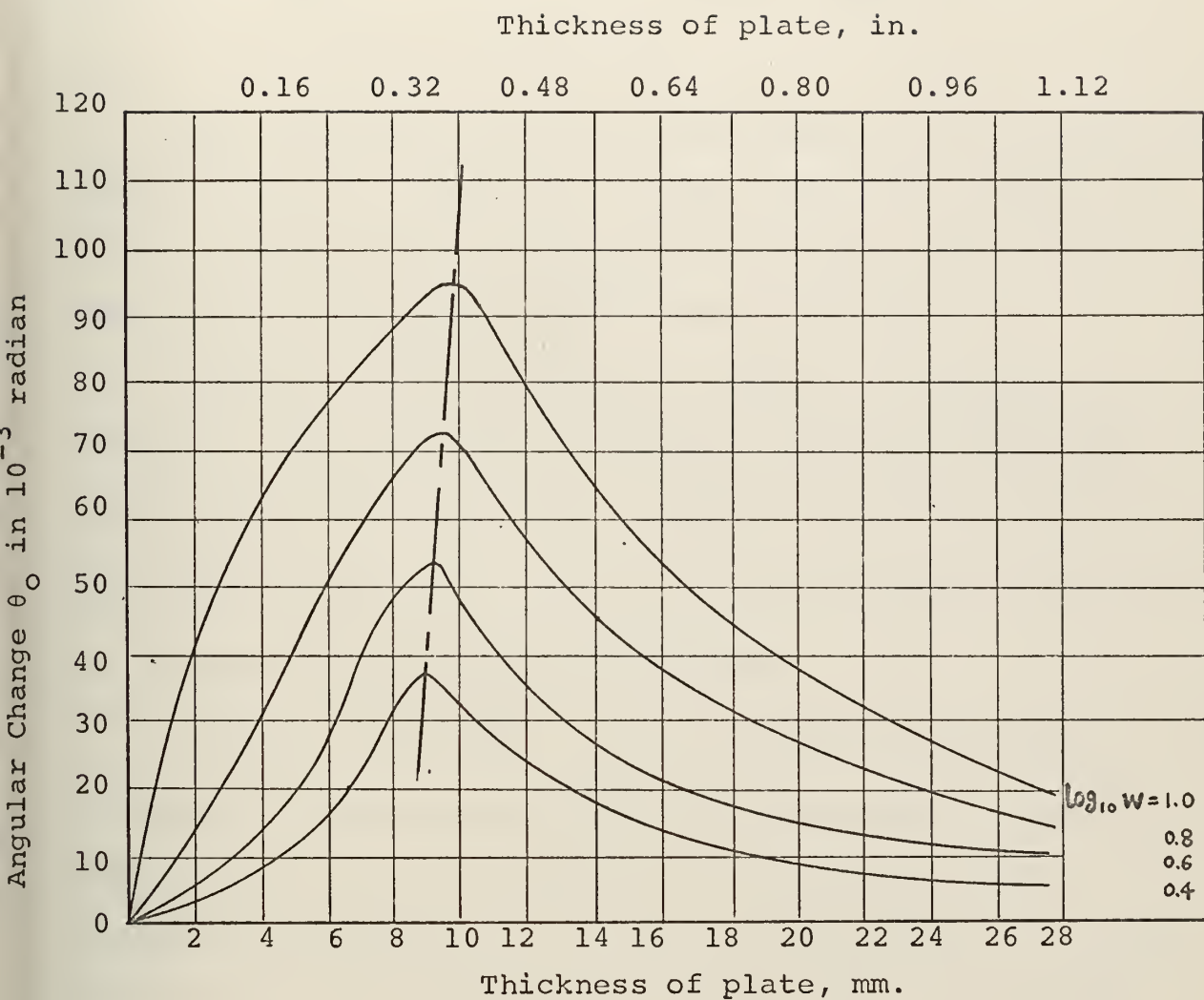


Figure 3 Variation of Angular Change of Free Fillet Welds, θ_0 , As a Function of Plate Thickness, t , and Weight of Electrode Consumed per Weld Length, W .

Masubuchi.⁽³⁾

$$C = t^4 / (1 + W/5) \quad \text{kg-mm/mm radian} \quad (8)$$

where W = weight of electrode consumed, gram/cm.

t = thickness of plate, mm.

The experimental results of determining the unconstrained angular change as a function of plate thickness, t (mm), and weight of electrode consumed per weld length, W (gr/cm), have been given by Hirai, et. al.⁽²⁾ as in Figure 3.

Therefore, for a given plate geometry and welding condition, the deflection of the one dimensional case can easily be estimated by using equations (1), (7), (8), and Figure 3.

C. Aim and Purpose of Present Studies

As stated previously, welding deformation has been analyzed only for the one dimensional case, but in reality most of the structural members are encountered in two dimensional deformation, in which the deformation, w , varies along the x - and y -directions as well as the unconstrained angular change, θ , and equivalent constrained welding moment, C . With the idea of previous investigations which state that the total strain energy in the plate during welding deformation becomes minimum, it is possible to extend the one dimensional deformation analysis to the two dimensional case. However, the simple mathematical solution using plate bending equation is rather difficult due to the uncertainty of the

equivalent load and the boundary conditions relevant to the welding deformation problem.

The difficulties can be easily overcome by formulating the problem into a finite element model because it can be self-adjusted to take care of the equivalent load and equivalent stiffness matrix due to welding. Furthermore, in this way the relation of deformation phenomena to the constrained angular change, θ , and the equivalent constrained welding moment C can be easily seen.

Therefore, it is intended to handle the two dimensional deformation with the finite element displacement technique.

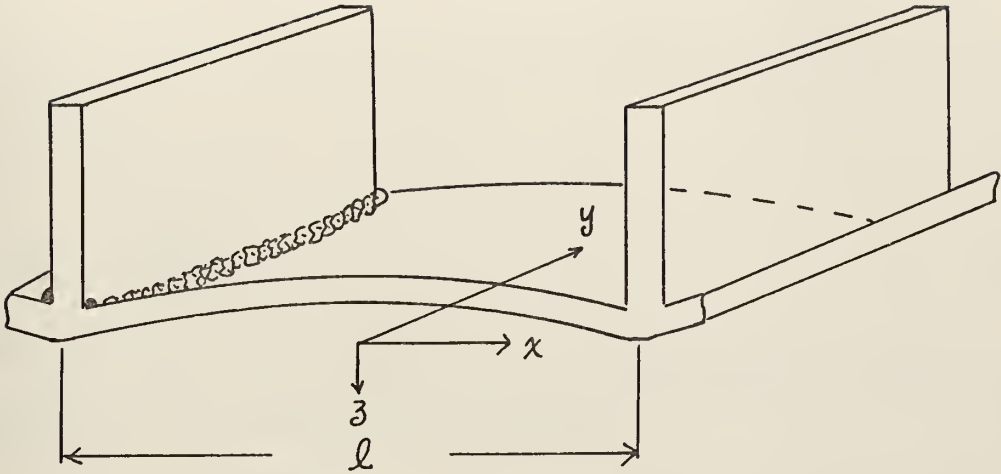
II FORMULATION OF FINITE ELEMENT EQUATION

A. One Dimensional Case

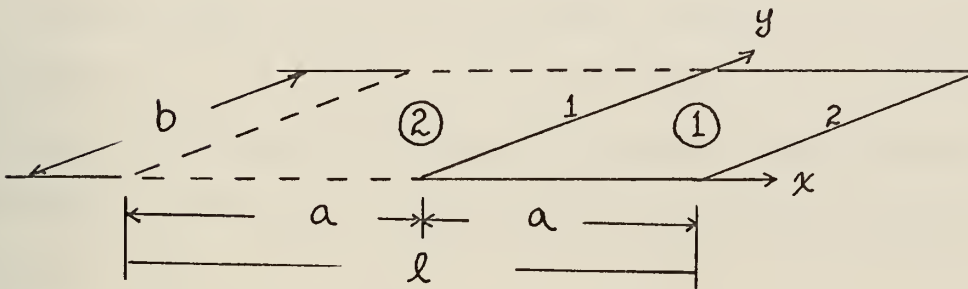
In this study, computer programs are programmed only for the two dimensional finite element analysis, but the one dimensional finite element formulations are presented to visualize the inside of the complex matrix assembling and computational procedure; furthermore, to compare the results to that of the previous investigators.

For the one dimensional case, as shown in Figure 2b, if deflections are elastic and small, the deflection can be calculated using elastic theory when the welding equivalent loads and the boundary conditions are prescribed. Even though the deformations are assumed to be elastic for the welded structural case, the loading and boundary conditions to be applied for this problem are not clear.

As introduced by Masubuchi, et. al.⁽¹⁾, if the concept of equivalent constrained welding moment, $C(\text{lb-in/in rad.})$ is being used, the uncertainty of loading conditions may be overcome; for the boundary condition it may be safe to assume to be simply supported for which only the angular change along the welded edge can be allowed. Furthermore, for simplicity of the problem, the deflections are considered only for the case of one span length of a simply supported beam to eliminate the complexity of interaction due to the statically indeterminate structural effects, as in Figure 4a.



a. Simplified one-span beam element



b. Local coordinate and nodal points for a finite beam element

Figure 4 One-span Beam Element and the Finite Element Local Coordinate

For the one dimensional plane stress, the strain energy can be expressed simply by:

$$U_p = 1/2 \iiint \epsilon_x \sigma_x dx dy dz \quad (9)$$

Using the stress-strain and strain-curvature relation for the beam element,

$$\sigma_x = E \epsilon_x \quad (10a)$$

$$\epsilon_x = -z \frac{d^2 w}{dx^2} \quad (10b)$$

Therefore, the strain energy can be expressed in terms of deflection and rigidity of the plate:

$$U_p = \frac{EI}{2} \int_0^a \left(\frac{d^2 w}{dx^2} \right)^2 dx \quad (11)$$

where

$$I = 2 \int_0^{t/2} \int_0^b z^2 dz dy = \frac{bt^3}{12}$$

On the other hand, the welding energy can be expressed in terms of θ_o , θ , and C , as in equation (5). For this problem it is more convenient to express the welding energy in terms of deflection rather than the angular changes. From equation (5),

$$U_w = \int_0^a \int_0^b \frac{c}{2} (\theta_o - \theta)^2 dy dx$$

the constrained angular change can be expressed by the equation

$$\theta = \partial w / \partial x \quad (12)$$

Therefore, the welding energy expressed,

$$U_w = \frac{bc}{2} \int_0^a \left(\theta_0 - \frac{\partial w}{\partial x} \right)^2 dx \quad (13)$$

where the values of C and θ_0 are constant which will be determined by the particular welding condition and the geometry.

The equilibrium condition for the system requires that the total strain energy has to be minimized, in other words, the variation of the total strain energy has to be zero, which becomes

$$\delta U_T = \delta U_p + \delta U_w = 0 \quad (14)$$

using equations (11) and (13), equation (14) becomes

$$\delta U_T = \delta \left[\frac{EI}{2} \int_0^a \left(\frac{d^2 w}{dx^2} \right)^2 dx \right] + \delta \left[\frac{bc}{2} \int_0^a \left(\theta_0 - \frac{dw}{dx} \right)^2 dx \right] = 0$$

But the variation of the constant term is zero, therefore,

$$EI \int_0^a \left(\frac{d^2 \delta w}{dx^2} \right) \left(\frac{d^2 w}{dx^2} \right) dx + \frac{bc}{2} \int_0^a \left(\frac{d \delta w}{dx} \right) \left(\frac{dw}{dx} \right) dx - bc \int_0^a \left(\frac{d \delta w}{dx} \right) dx \quad (15)$$

For a beam element as shown in Figure 4b, the deflection can be assumed to be a general cubic function of x ,

$$w = c_1 + c_2 x + c_3 x^2 + c_4 x^3 \quad (16)$$

and when the nodal displacements and the nodal angles are the unknown variables which are to be calculated, the coefficients (c_1, c_2, c_3, c_4) in equation (16) can exclusively be expressed in terms of the nodal displacements and angles considering the requirements of continuity at the nodal points

$$w(x=0) = w_1 = c_1$$

$$\frac{\partial w}{\partial x}(x=0) = w_{x_1} = c_2$$

$$w(x=a) = w_2 = c_1 + c_2 a + c_3 a^2 + c_4 a^3$$

$$\frac{\partial w}{\partial x}(x=a) = w_{x_2} = c_2 + 2c_3 a + 3c_4 a^2$$

The above requirements can be expressed in the matrix form,

$$\underline{q} = \underline{T} \underline{C} \quad (17)$$

where

$$\underline{T} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & a & a^2 & a^3 \\ 0 & 1 & 2a & 3a^2 \end{pmatrix} \quad (17a)$$

$$\underline{C} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} \quad (17b) \quad \text{and} \quad \underline{q} = \begin{pmatrix} w_1 \\ w_{x_1} \\ w_2 \\ w_{x_2} \end{pmatrix} \quad (17c)$$

From equation (17), \underline{C} can be expressed

$$\underline{C} = \underline{T}^{-1} \underline{q} \quad (18)$$

If the new matrix \underline{G} is defined such that

$$\underline{G} = \underline{T}^{-1} \quad (19)$$

then,

$$\underline{C} = \underline{G} \underline{q} \quad (20)$$

where

$$\underline{\underline{G}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3/a^2 & -2/a & 3/a^2 & -1/a \\ 2/a^3 & 1/a^2 & -2/a^3 & 1/a^2 \end{pmatrix} \quad (21)$$

The second derivative of deflection function becomes,

$$\frac{d^2 w}{dx^2} = 2c_3 + 6c_4 x$$

which is in matrix form

$$\frac{d^2 w}{dx^2} = \underline{\underline{H}} \underline{\underline{C}} \quad (22)$$

where $\underline{\underline{H}}$ is the matrix defined by

$$\underline{\underline{H}} = (0 \quad 0 \quad 2 \quad 6x) \quad (23)$$

then,

$$\left(\frac{d^2 \delta w}{dx^2} \right) \left(\frac{d^2 w}{dx^2} \right) = \delta (\underline{\underline{H}} \underline{\underline{C}})^T (\underline{\underline{H}} \underline{\underline{C}}) = (\delta \underline{\underline{C}}^T) (\underline{\underline{H}}^T \underline{\underline{H}} \underline{\underline{C}})$$

but, using the equation (20),

$$\left(\frac{d^2 \delta w}{dx^2} \right) \left(\frac{d^2 w}{dx^2} \right) = (\delta \underline{\underline{q}}^T) (\underline{\underline{G}}^T \underline{\underline{H}}^T \underline{\underline{H}} \underline{\underline{G}} \underline{\underline{q}}) \quad (24)$$

The matrix expression for the first derivative of the deflection w can be expressed in the same manner,

$$\frac{dw}{dx} = c_2 + 2c_3 x + 3c_4 x^2$$

using $\underline{\underline{F}}$ and $\underline{\underline{C}}$

$$\frac{dw}{dx} = \underline{\underline{F}} \underline{\underline{C}} \quad (25)$$

where

$$\underline{F} = (0 \quad 1 \quad 2x \quad 3x^2) \quad (26)$$

Finally, using equation (20)

$$\frac{d\delta w}{dx} = (\delta \underline{q}^T) (\underline{G}^T \underline{F}^T) \quad (27)$$

and

$$\left(\frac{d\delta w}{dx} \right) \left(\frac{dw}{dx} \right) = (\delta \underline{q}^T) (\underline{G}^T \underline{F}^T \underline{F} \underline{G} \underline{q}) \quad (28)$$

From equations (24), (27) and (28), the minimum energy equation (15) can be written in the matrix form,

$$\begin{aligned} EI \int_0^a (\delta \underline{q}^T) (\underline{G}^T \underline{H}^T \underline{H} \underline{G} \underline{q}) dx \\ + bc \int_0^a (\delta \underline{q}^T) (\underline{G}^T \underline{F}^T \underline{F} \underline{G} \underline{q}) dx \\ - bc \int_0^a (\delta \underline{q}^T) (\underline{G}^T \underline{F}^T) dx = 0 \end{aligned} \quad (29)$$

Noticing that the matrix \underline{q} , \underline{q}^T , \underline{G} , and \underline{G} is not the function of x , but only of the function of dimension of the plate, and displacement and angle at the nodal points; therefore, equation (29) can be simplified to

$$\begin{aligned} \delta \underline{q}^T \{ EI \underline{G}^T \left[\int_0^a (\underline{H}^T \underline{H}) dx \right] \underline{G} \underline{q} \\ + bc \underline{G}^T \left[\int_0^a (\underline{F}^T \underline{F}) dx \right] \underline{G} \underline{q} \\ - bc \underline{G}^T \left[\int_0^a (\underline{F}^T) dx \right] \} = 0 \end{aligned} \quad (30)$$

in which the only integrations necessary to be carried out are the bracketed quantities, which become after integration,

$$\int_0^a (\tilde{H}^T \tilde{H}) dx = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 4a & 6a^2 \\ 0 & 0 & 6a^2 & 12a^3 \end{pmatrix} \quad (31)$$

$$\int_0^a (\tilde{F}^T \tilde{F}) dx = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 2a & 3a^2 \\ 0 & 2a & 4a^2 & 6a^3 \\ 0 & 3a^2 & 6a^3 & 9a^4 \end{pmatrix} \quad (32)$$

$$\int_0^a (\tilde{F}^T) dx = \begin{pmatrix} 0 \\ 1 \\ 2a \\ 3a^2 \end{pmatrix} \quad (33)$$

Equation (30) is the final equation for one element formulated by the finite element displacement method. The first quantity is defined as a stiffness matrix for an element,⁽¹¹⁾ the second is the additional stiffness matrix due to welding, and the third is the equivalent load due to welding. They are as follows:

Stiffness matrix for an element,

$$\begin{aligned} \underline{k} &= EI \underline{G}^T \left[\int_0^a (\underline{H}^T \underline{H}) dx \right] \underline{G} \\ &= EI \begin{pmatrix} 12/a^3 & 6/a^2 & -12/a^3 & 6/a^2 \\ 6/a^2 & 4/a & -6/a^2 & 2/a \\ -12/a^3 & -6/a^2 & 12/a^3 & -6/a^2 \\ 6/a^2 & 2/a & -6/a^2 & 4/a \end{pmatrix} \end{aligned} \quad (34)$$

Additional stiffness matrix due to welding,

$$\begin{aligned} \underline{k}_w &= bc \underline{G}^T \left[\int_0^a (\underline{F}^T \underline{F}) dx \right] \underline{G} \\ &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & bc \end{pmatrix} \end{aligned} \quad (35)$$

Load matrix due to welding,

$$\begin{aligned} Q &= bc \underline{G}^T \left[\int_0^a \underline{F}^T dx \right] \\ &= \begin{pmatrix} 0 \\ 0 \\ 0 \\ bc\theta_o \end{pmatrix} \end{aligned} \quad (36)$$

At this stage, assembling of equation (30) has to be considered when the one-span beam is divided by many finite elements, but the one-span beam element of Figure 4a is chosen to be two finite elements as in Figure 4b; then, only element 1 can be used to predict the deflection and angular

change at the nodal points by symmetric argument. In this case, the simultaneous equations to be solved are (from equation (30)),

$$\delta w_1 \left[\frac{6EI}{a^2} \left(\frac{2}{a} w_1 + w_{x_1} - \frac{2}{a} w_2 + w_{x_2} \right) \right] = 0$$

$$\delta w_{x_1} \left[\frac{2EI}{a} \left(\frac{3}{a} w_1 + 2w_{x_1} - \frac{3}{a} w_2 + w_{x_2} \right) \right] = 0$$

$$\delta w_2 \left[\frac{6EI}{a^2} \left(-\frac{2}{a} w_1 - w_{x_1} + \frac{2}{a} w_2 - w_{x_2} \right) \right] = 0$$

$$\delta w_{x_2} \left[\frac{2EI}{a} \left(\frac{3}{a} w_1 + w_{x_1} - \frac{3}{a} w_2 + \left(2 + \frac{abc}{2EI} \right) w_{x_2} - bc\theta_o \right) \right] = 0$$

(37a-d)

From the simply supported boundary condition at node 2, $w_2 = 0$; therefore, $\delta w_2 = 0$.

From symmetry condition at node 1, $w_{x_1} = 0$; therefore, $\delta w_{x_1} = 0$.

Furthermore, the displacement at node 1 and the angle at node 2 is not known, which means that the variations of these variables are not zero; therefore, the only way to satisfy equation (37a) and (37d) is the bracketed value in each equation has to be zero, which leads to final equation for w and w_{x_2} ,

$$\frac{2}{a} w_1 + w_{x_2} = 0$$

$$\frac{3}{a} w_1 + \left(2 + \frac{abc}{2EI} \right) w_{x_2} = 0 \quad (38a-b)$$

From equation (38a-b), displacement at the middle point and angular change at the edge expressed as,

$$w_1 = \frac{-a\theta_0}{2(1 + EI/abc)} \quad (39)$$

$$w_{x_2} = \frac{\theta_0}{2(1 + EI/abc)} \quad (40)$$

It is worth to compare the results of equations (6) and (7) with equations (39) and (40). In equations (39) and (40),

$$\frac{EI}{abc} = \frac{bt^3/12}{abc} = \frac{t^3/12}{ac}$$

and $a = \ell/2$ is to be used. Then,

$$w_1 = \frac{-\ell\theta_0}{4(1 + 2EI/\ell c)} \quad (41)$$

$$w_{x_2} = \frac{\theta_0}{1 + 2EI/\ell c} \quad (42)$$

where I is defined by unit width basis, ^(1,6) which is $I = t^3/12$.

Equation (42) agrees with equation (7), as also does equation (41), if $x = \ell/2$ and θ of equation (7) is plugged into equation (1). The negative sign of equation (41) means that the coordinate and the positive angle at the edge has to be chosen, as in Figure 4a.

As shown, the results from the finite element method is exactly the same expression proposed by Masubuchi, et.al. ^(1,3) for the one dimensional deformation, and in this way, the equivalent loads and the stiffness matrix due to welding become easily visualized.

B. Two Dimensional Case

1. Formulation of the Problem

In the two dimensional analysis, all procedures are the same as that of the one dimensional analysis except that the integrations are carried out by the numerical methods and the methods of assembling are mentioned. The deflection may be approximated by the bi-cubic function for one element, ^(12,14) as in Figure 5.

$$\begin{aligned}
 w = & c_1 + c_2x + c_3y + c_4xy + c_5x^2 + c_6y^2 + c_7x^3 \\
 & + c_8x^2y + c_9xy^2 + c_{10}y^3 + c_{11}x^3y + c_{12}x^2y^2 \\
 & + c_{13}xy^3 + c_{14}x^3y^2 + c_{15}x^2y^3 + c_{16}x^3y^3
 \end{aligned} \tag{43a}$$

$$\begin{aligned}
 \frac{\partial w}{\partial x} = & c_2 + c_4y + 2c_5x + 3c_7x^2 + 2c_8xy + c_9y^2 \\
 & + 3c_{11}x^2y + 2c_{12}xy^2 + c_{13}y^3 + 3c_{14}x^2y^2 \\
 & + 2c_{15}xy^3 + 3c_{16}x^2y^3
 \end{aligned} \tag{43b}$$

$$\begin{aligned}
 \frac{\partial w}{\partial y} = & c_3 + c_4x + 2c_6y + c_8x^2 + 2c_9xy + 3c_{10}y^2 \\
 & + c_{11}x^3 + 2c_{12}x^2y + 3c_{12}xy^2 + 2c_{14}x^3y \\
 & + 3c_{15}x^2y^2 + 3c_{16}x^3y^2
 \end{aligned} \tag{43c}$$

$$\begin{aligned}
 \frac{\partial^2 w}{\partial x \partial y} = & c_4 + 2c_8x + 2c_9y + 3c_{11}x^2 + 4c_{12}xy \\
 & + 3c_{13}y^2 + 6c_{14}x^2y + 6c_{15}xy^2 + 9c_{16}x^2y^2
 \end{aligned} \tag{43d}$$

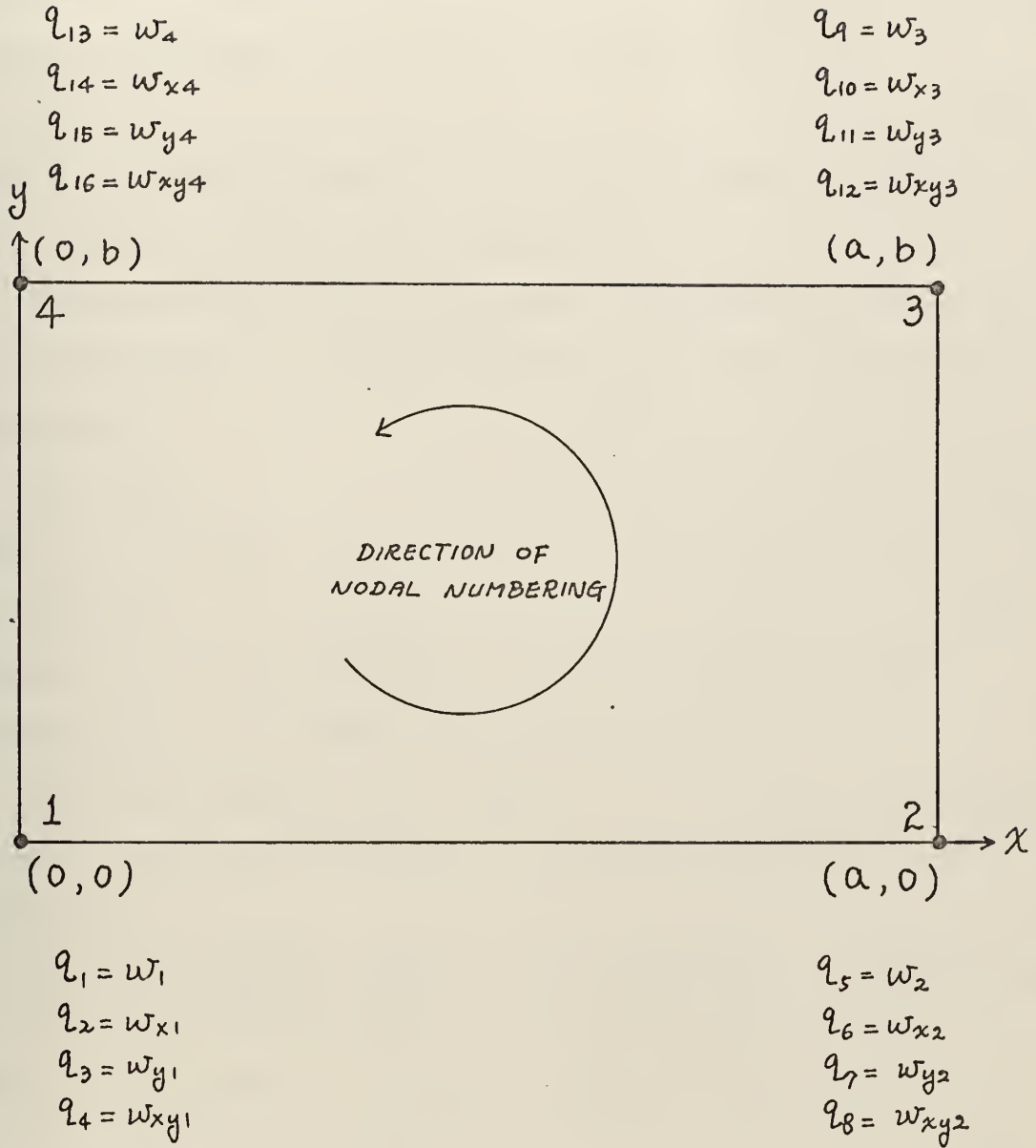


Figure 5 Local Coordinate of a Two dimensional Rectangular Finite Element and Generalized Displacement Numbering

From equation (43a-d), the matrix equation can be defined such that at each nodal point, the generalized displacements (deflections, angles in x and y, and twisting angles) are expressed in terms of the geometry of the element and the some unknown coefficients C_i , which is the form of,

$$\underline{q} = \underline{T} \underline{C} \quad (44)$$

where \underline{T} is 16 by 16 matrix, \underline{C} is 16 by 1 matrix, and \underline{q} is 16 by 1 matrix defined as in Appendix A.

From equation (44), the unknown coefficient matrix, \underline{C} , can be expressed in terms of geometry and the generalized displacements,

$$\underline{C} = \underline{G} \underline{q} \quad (45)$$

where

$$\underline{G} = \underline{T}^{-1} \quad (46)$$

However, the actual calculation of the inverse of \underline{T} is carried out by the computer in this analysis.

From the elastic plate theory, ^(8,9,10) the strain energy stored in the plate during the elastic deformation is expressed by

$$U_p = \frac{D}{2} \int_0^a \int_0^b \left\{ \left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right]^2 - 2(1-\nu) \left[\frac{\partial^2 w}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x \partial y} \right]^2 \right\} dx dy$$

This can be written in the matrix form, ⁽⁷⁾ which is

$$U_p = \frac{1}{2} \int_0^a \int_0^b \underline{K}^T \underline{D} \underline{K} dy dx \quad (47)$$

where

$$\underline{\underline{K}} = - \begin{pmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2 \frac{\partial^2 w}{\partial x \partial y} \end{pmatrix} = -\underline{\underline{H}} \underline{\underline{C}} \quad (48)$$

in which $\underline{\underline{H}}$ is given by Appendix A,

$$\underline{\underline{D}}' = \frac{Et^3}{12(1-\nu^2)} \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix} = \beta \cdot \underline{\underline{D}} \quad (49)$$

in which

$$\beta = \frac{Et^3}{12(1-\nu^2)}$$

and

$$\underline{\underline{D}} = \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix}$$

Therefore, equation (48) becomes after taking the variation with respect to generalized displacement,

$$\begin{aligned} \delta U_p &= \int_0^a \int_0^b (\delta \underline{\underline{K}}^T) \beta \underline{\underline{D}} \underline{\underline{K}} dy dx \\ &= \int_0^a \int_0^b (\delta \underline{\underline{C}}^T) (\underline{\underline{H}}^T \beta \underline{\underline{D}} \underline{\underline{H}} \underline{\underline{C}}) dy dx \\ &= \delta \underline{\underline{q}}^T \beta \underline{\underline{G}}^T \left[\int_0^a \int_0^b \underline{\underline{H}}^T \underline{\underline{D}} \underline{\underline{H}} dy dx \right] \underline{\underline{G}} \underline{\underline{q}} \end{aligned} \quad (50)$$

The welding energy for the elements shown in Figure 6b can be written as,

$$U_w = \int_0^a \frac{C_y}{2} (\theta_o - \theta_y)^2 dx + \int_0^b \frac{C_x}{2} (\theta_o - \theta_x)^2 dy$$

Therefore,

$$U_w = \frac{C_y}{2} \int_0^a \left(\theta_o - \frac{\partial w}{\partial y} \right)^2 dx + \frac{C_x}{2} \int_0^b \left(\theta_o - \frac{\partial w}{\partial x} \right)^2 dy \quad (51)$$

which becomes,

$$\begin{aligned} U_w = & \left[\frac{C_y}{2} \int_0^a \left(\frac{\partial w}{\partial y} \right)^2 dx + \frac{C_x}{2} \int_0^b \left(\frac{\partial w}{\partial x} \right)^2 dy \right] \\ & - \left[C_y \theta_o \int_0^a \frac{\partial w}{\partial y} dx + C_x \theta_o \int_0^b \frac{\partial w}{\partial x} dy \right] \\ & + \left[\frac{1}{2} (C_y + C_x) \theta_o^2 \right] \end{aligned} \quad (52)$$

and taking the variation,

$$\begin{aligned} \delta U_w = & \left[C_y \int_0^a \left(\frac{\partial \delta w}{\partial y} \right) \left(\frac{\partial w}{\partial y} \right) dx + C_x \int_0^b \left(\frac{\partial \delta w}{\partial x} \right) \left(\frac{\partial w}{\partial x} \right) dy \right] \\ & - \left[C_y \theta_o \int_0^a \left(\frac{\partial \delta w}{\partial y} \right) dx + C_x \theta_o \int_0^b \left(\frac{\partial \delta w}{\partial x} \right) dy \right] \end{aligned} \quad (53)$$

in which the first bracketed term is the additional stiffness matrix due to welding, and the second is equivalent load matrix for the welding. In matrix form, equation (53) can be written, if some matrices defined as below:

$$\frac{\partial w}{\partial x} = \underset{\sim}{F}_y \underset{\sim}{C} = \underset{\sim}{C}^T \underset{\sim}{F}_y^T \quad (54a)$$

$$\frac{\partial w}{\partial y} = \underline{\underline{F}}_x \underline{\underline{C}} = \underline{\underline{C}}^T \underline{\underline{F}}_x^T \quad (54b)$$

where the subscripts x and y denote that the integration should be carried out along the x- and y-direction, respectively, and the matrix $\underline{\underline{F}}_x$ and $\underline{\underline{F}}_y$ are defined in Appendix A. Therefore,

$$\left(\frac{\partial \delta w}{\partial y} \right) \left(\frac{\partial w}{\partial y} \right) = (\delta \underline{\underline{C}}^T) (\underline{\underline{F}}_y^T \underline{\underline{F}}_y \underline{\underline{C}}) = (\delta \underline{\underline{q}}^T) \underline{\underline{G}}^T \underline{\underline{F}}_y^T \underline{\underline{F}}_y \underline{\underline{G}} \underline{\underline{q}} \quad (55a)$$

$$\left(\frac{\partial \delta w}{\partial x} \right) \left(\frac{\partial w}{\partial x} \right) = (\delta \underline{\underline{C}}^T) (\underline{\underline{F}}_x^T \underline{\underline{F}}_x \underline{\underline{C}}) = (\delta \underline{\underline{q}}^T) \underline{\underline{G}}^T \underline{\underline{F}}_x^T \underline{\underline{F}}_x \underline{\underline{G}} \underline{\underline{q}} \quad (55b)$$

Using equations (54a-b) and (55a-b), the welding term can be written as:

$$\begin{aligned} \delta U_w = & \delta \underline{\underline{q}}^T \underline{\underline{C}}_y \underline{\underline{G}}^T \left[\int_0^a (\underline{\underline{F}}_x^T \underline{\underline{F}}_x) dx \right] \underline{\underline{G}} \underline{\underline{q}} \\ & + \delta \underline{\underline{q}}^T \underline{\underline{C}}_x \underline{\underline{G}}^T \left[\int_0^b (\underline{\underline{F}}_y^T \underline{\underline{F}}_y) dy \right] \underline{\underline{G}} \underline{\underline{q}} \\ & - \delta \underline{\underline{q}}^T \underline{\underline{C}}_y \theta_o \underline{\underline{G}}^T \left[\int_0^a (\underline{\underline{F}}_x^T) dx \right] \\ & - \delta \underline{\underline{q}}^T \underline{\underline{C}}_x \theta_o \underline{\underline{G}}^T \left[\int_0^b (\underline{\underline{F}}_y^T) dy \right] \end{aligned} \quad (56)$$

As in the one dimensional case already shown, the total variation should be zero to be in equilibrium of this system. Therefore,

$$\delta U_T = \delta U_P + \delta U_w = 0$$

which is from equations (50) and (56),

$$\begin{aligned}
 (\delta \underline{q}^T) \left\{ \underline{G}^T \left[\beta \int_0^a \int_0^b \underline{H}^T \underline{D} \underline{H} \, dy \, dx + C_Y \int_0^a \underline{F}_x^T \underline{F}_x \, dx \right. \right. \\
 \left. \left. + C_x \int_0^b \underline{F}_y^T \underline{F}_y \, dy \right] \underline{G} \underline{q} - \underline{G}^T [C_Y \theta_0 \int_0^a \underline{F}_x^T \, dx \right. \\
 \left. + C_x \theta_0 \int_0^b \underline{F}_y^T \, dy] \right\} = 0
 \end{aligned} \quad (57)$$

These are the 16 by 16 simultaneous equations which have to be solved if only one element is concerned as shown in the one dimensional case, with the proper boundary and symmetry conditions. From equation (57), the stiffness matrix for a plate element,

$$\underline{k} = \underline{G}^T \beta \int_0^a \int_0^b \underline{H}^T \underline{D} \underline{H} \, dy \, dx \underline{G} \quad (58a)$$

The additional stiffness matrix due to welding along the x-direction of the plate,

$$\underline{k}_{wx} = \underline{G}^T C_Y \int_0^b \underline{F}_x^T \underline{F}_x \, dx \underline{G} \quad (58b)$$

The additional stiffness matrix due to welding along the y-direction of the plate,

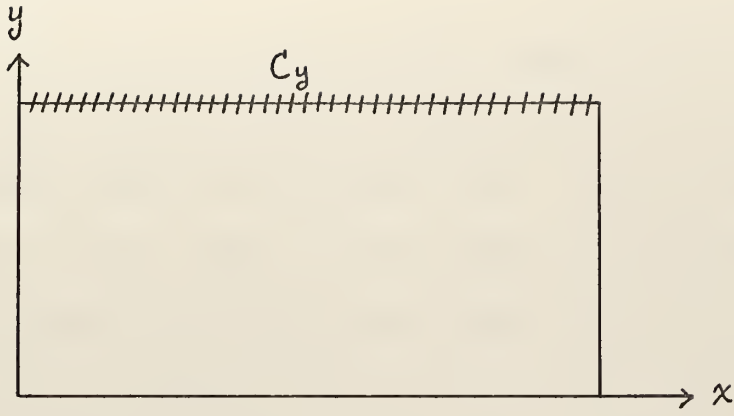
$$\underline{k}_{wy} = \underline{G}^T C_x \int_0^a \underline{F}_y^T \underline{F}_y \, dy \underline{G} \quad (58c)$$

The equivalent load matrix due to welding along the x-direction of the plate,

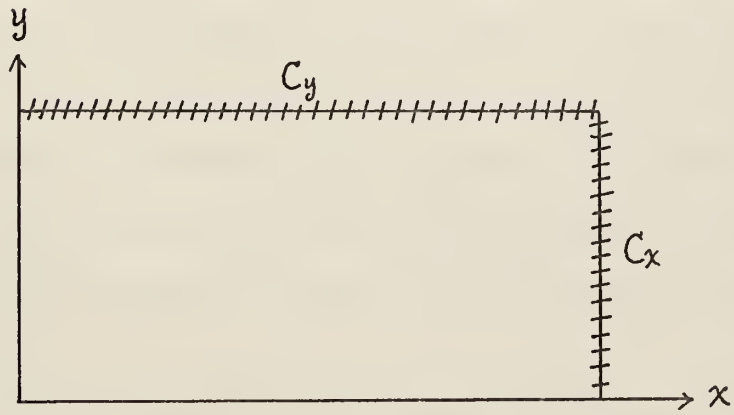
$$\underline{Q}_x = \underline{G}^T C_Y \theta_0 \int_0^a \underline{F}_x^T \, dx \quad (58d)$$

The equivalent load matrix due to welding along the y-direction of the plate,

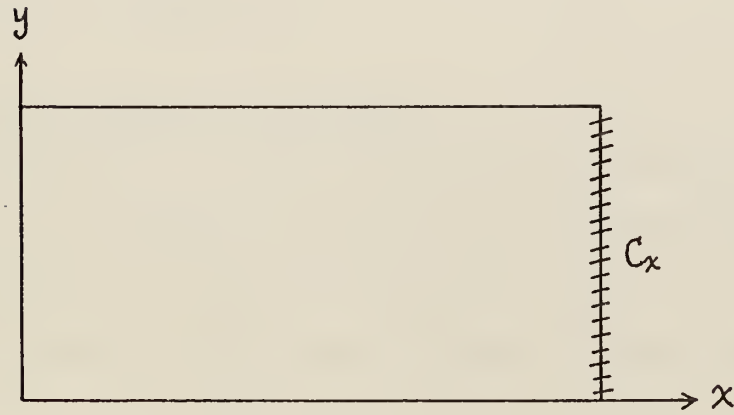
$$\underline{Q}_y = \underline{G}^T C_x \theta_0 \int_0^b \underline{F}_y^T \, dy \quad (58e)$$



(a)



(b)



(c)

Figure 6 Various Finite Element Having Welded Edge

2. Assembling Procedure

For all the finite elements except those having the welded edge, the stiffness matrix necessary to be assembled is the expression given by equation (58b), and for the finite elements having welded edges, three cases are involved. The first case is elements having welded edges along the x-direction as in Figure 6b; the second case is along the y-direction, as in Figure 6c; and the third case is along x- and y-directions as in Figure 6b. The assembling procedure is the expression of equation (57) for the overall plate which is to be solved, which is described in FEABL user's manual.⁽¹³⁾ Also, from the symmetry condition, which will be described in the following section, only one-quarter part of the plate is actually being considered.

Example of the assembling procedures for the two finite elements is given in Appendix B.

3. Symmetry and Boundary Conditions

After assembling, relevant symmetry and boundary conditions have to be applied to solve the simultaneous equations, as shown in the one dimensional case. Along the welded edge of x and y, the simply supported edge condition requires that all the displacements at that nodal point have to be zero, and for the x-directed edge, the angular changes in the x-direction are not allowed. In just the same way, for y-directed edge, the angular changes in y are not allowed. The symmetry condition requires that the first derivative

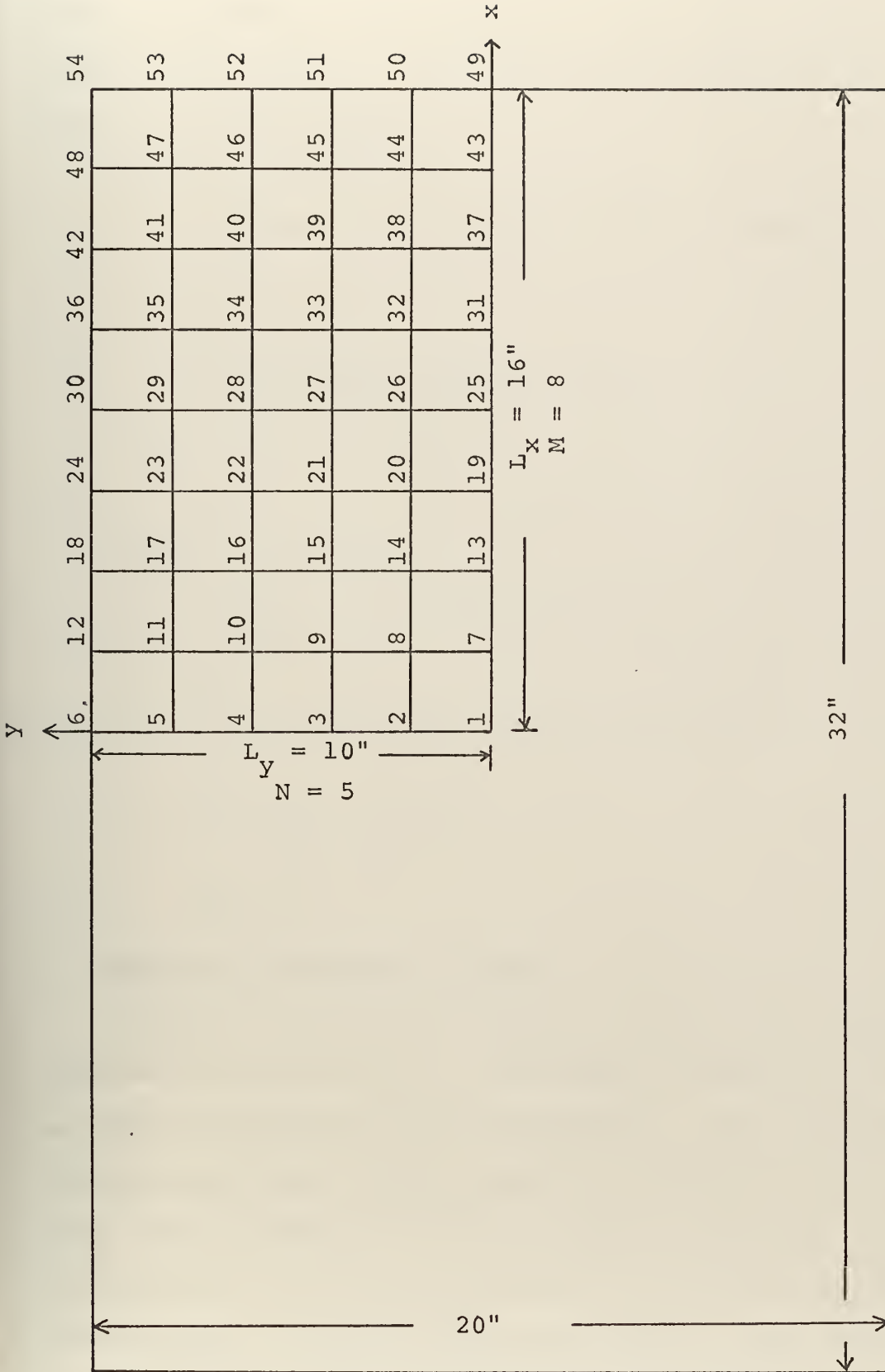


Figure 7 Dimensions and Element Meshes of One-quarter of the Plate for Computer Input

along the symmetric axis should be zero. For example, the symmetry and boundary conditions for the case given by Figure 7 are as follows:

Boundary conditions:

At node 6, 12, 18, 24, 30, 36, 42, 48, 54

$$w = 0$$

$$w_x = 0$$

At node 49, 50, 51, 52, 53, 54

$$w = 0$$

$$w_y = 0$$

Symmetry conditions:

At node 1, 2, 3, 4, 5, 6

$$w_y = 0$$

At node 1, 7, 13, 19, 25, 31, 37, 43

$$w_x = 0$$

At node 1, 6, 49

$$w_{xy} = 0.$$

4. Numerical Integration Method

Usually in the finite element analysis, the integrations involved are carried out by the numerical methods, which can be easily calculated using the computer. For the present analysis, the nine Gaussian points are chosen for the rectangular finite element, ^(14,18) as shown in Figure 8c. In actual integration, it is very convenient to use the normalized coordinate system, as will be described below. From Figure 8a and 8b, normalized coordinate system can be defined as

$$x = \frac{a}{2}(X + 1) \quad (59a)$$

$$y = \frac{b}{2}(Y + 1) \quad (59b)$$

and

$$dx = \frac{a}{2} dX \quad (60a)$$

$$dy = \frac{b}{2} dY \quad (60b)$$

Therefore, the arbitrary integration

$$f(a,b) = \int_0^a \int_0^b P(x,y) dx dy \quad (61)$$

becomes in the normalized coordinates,

$$f(a,b) = \frac{ab}{4} \int_{-1}^1 \int_{-1}^1 P\left[\frac{a}{2}(X+1), \frac{b}{2}(Y+1)\right] \quad (62)$$

Equation (62) can be integrated by numerically using

9-Gaussian points, which become

$$f(a,b) = \frac{ab}{4} \sum_{i=1}^3 \sum_{j=1}^3 W_i W_j P(x,y) \quad (63)$$

where

$$x = \frac{a}{2}(X_i + 1) \quad (63a)$$

$$y = \frac{b}{2}(Y_j + 1) \quad (63b)$$

$$W_i: W_1 = 5/9, W_2 = 8/9, W_3 = 5/9 \quad (63c)$$

$$W_j: W_1 = 5/9, W_2 = 8/9, W_3 = 5/9 \quad (63d)$$

$$X_i: X_1 = -\sqrt{15}/5, X_2 = 0, X_3 = \sqrt{15}/5 \quad (63e)$$

$$Y_j: Y_1 = -\sqrt{15}/5, Y_2 = 0, Y_3 = \sqrt{15}/5 \quad (63f)$$

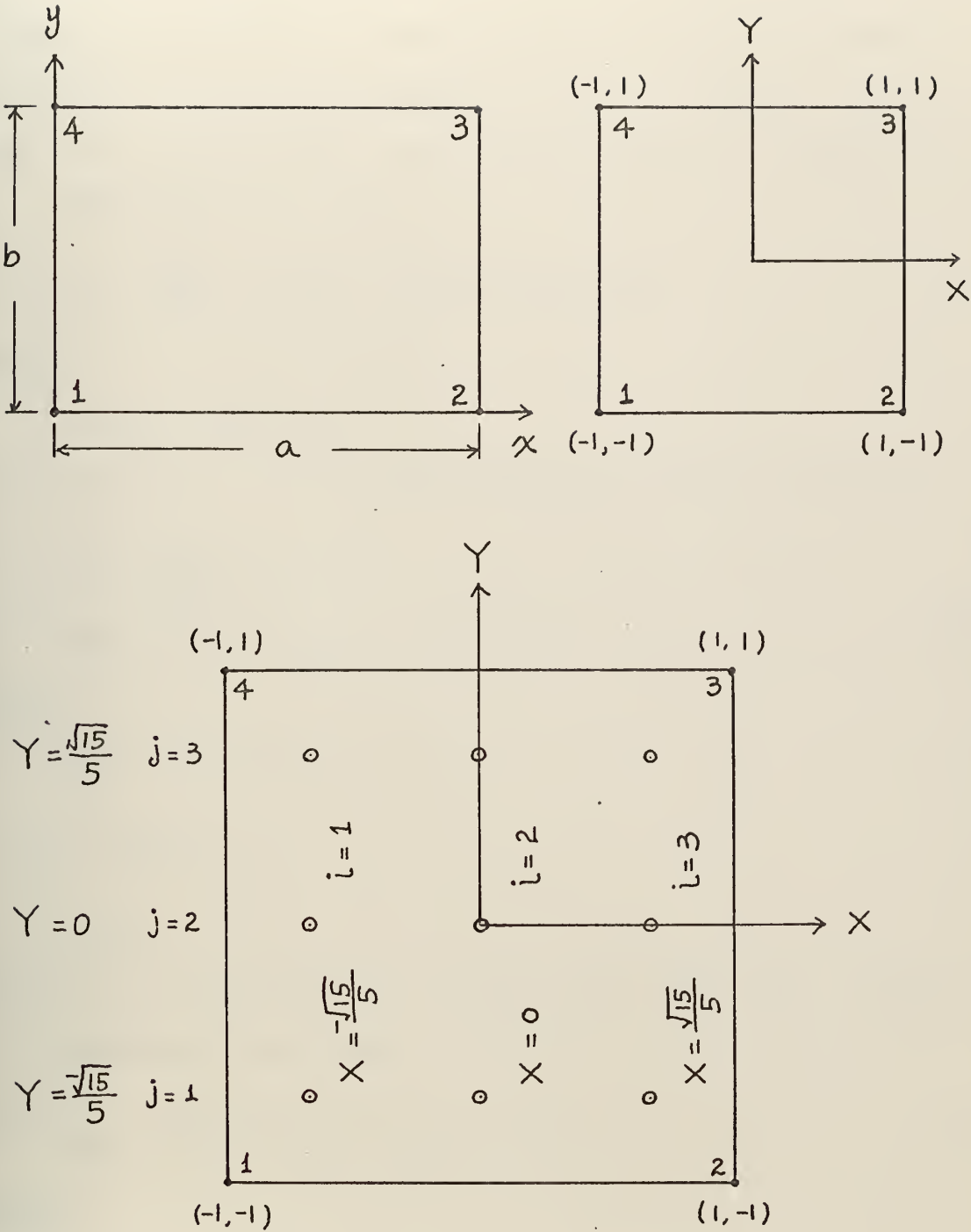


Figure 8 Ordinally and Normalized Coordinates for an Element, and 9 Gaussian Points for Numerical Integration

Therefore, using the concept of normalizing and numerical integration described above, the integrations involved in the present analysis such as equation (58a-e) can easily be written, which are programmed in Appendix D.

From equation (58a)

$$\underline{k} = \left(\frac{ab\beta}{4}\right) \underline{G}^T \left[\sum_{i=1}^3 \sum_{j=1}^3 W_i W_j \underline{P}(x,y) \right] \underline{G} \quad (64)$$

where

$$\underline{P}(x,y) = \underline{H}^T \underline{D} \underline{H}$$

x , y , W_i , W_j , X_i and Y_j are defined by equation (63a-f).

From equation (58b),

$$\underline{k}_{wx} = \frac{aC_y}{2} \underline{G}^T \left[\sum_{i=1}^3 W_i \underline{P}_x(x,y) \right] \underline{G} \quad (65)$$

where

$$\underline{P}_x(x,y) = \underline{F}_x^T \underline{F}_x$$

$$x = \frac{a}{2}(X_i + 1)$$

$$y = b$$

W_i and X_i are defined by equation (63c) and (63e), respectively.

From equation (58c),

$$\underline{k}_{wy} = \frac{bC_x}{2} \underline{G}^T \left[\sum_{j=1}^3 W_j \underline{P}_y(x,y) \right] \underline{G} \quad (66)$$

where

$$\underline{P}_y(x,y) = \underline{F}_y^T \underline{F}_y$$

$$x = a$$

$$y = \frac{b}{2}(Y_j + 1)$$

W_j and X_j are defined by equations (63d) and (63f), respectively.

From equation (58d),

$$\underline{Q}_x = \frac{a\theta_o C_y}{2} \underline{G}^T \left[\sum_{i=1}^3 W_i \underline{P}_{\ell x}(x, y) \right] \underline{G} \quad (67)$$

where

$$\underline{P}_{\ell x}(x, y) = \underline{F}_x^T$$

$$x = \frac{a}{2}(X_i + 1)$$

$$y = b$$

W_i and X_i are defined by equation (63c) and (63), respectively.

From equation (58e),

$$\underline{Q}_y = \frac{b\theta_o C_x}{2} \underline{G}^T \left[\sum_{j=1}^3 W_j \underline{P}_{\ell y}(x, y) \right] \underline{G} \quad (68)$$

where

$$\underline{P}_{\ell y}(x, y) = \underline{F}_y^T$$

$$x = a$$

$$y = \frac{b}{2}(Y_j + 1)$$

W_j and Y_j are defined by equations (63d) and (63f).

As noticed above, numerical integration using the normalized coordinate can simplified the computing procedure by factoring out a and b .

5. Computer Programs

Computations of the ordinally, additional welding, and equivalent welding load matrix of equation (58a-e) have been programmed by using the numerical integration technique with normalized coordinates as listed in Appendix D. In addition to these, FEABL Programs⁽¹³⁾ in assembling of the relevant matrices and solving the simultaneous equations with proper boundary and symmetry conditions are also used and listed in Appendix D.

The Programs listed in Appendix D are only relevant to the panel structure welding with rectangular finite elements. Further, the nodal numbering system should follow the way given by the sample analysis, which means that the numbering should start from origin to y-direction, as shown in Figure 7.

Input data for this analysis are the overall dimensions of one panel structure in the x- and y-direction, thickness of the plate, Young's modulus of elasticity, Poisson's ratio, number of elements divided in x- and y-direction, equivalent welding moment in x- and y-direction, and free joint angular change at the edge. The input data cards are described in Appendix C.

Outputs are the displacements, angular changes in x- and y-directions, and the twisting angles at each nodal point, as shown in Appendix E.

One thing to be noticed here is that for a quarter part of the plate, as shown in Figure 9, the proper boundary and symmetry conditions at the nodal points are to be generated

by subroutine HOLD given in Appendix D; therefore, as long as boundary conditions of simply supported and symmetry conditions as described are concerned, it is not necessary to be considered in the input. Therefore, no matter how the number of finite elements are to be chosen, the only input data will be as described in equation (69a-h). Furthermore, to use this program, the coordinate and a quarter part of the plate should be chosen as the same way as shown in Figure 9.

The units used in the sample program are pounds, inches, and radians. As results, for the output data, the deflections are in inches, the first derivatives (angular changes) are in radians, and the second derivatives with respect to x and y (twisting angles) are also in radians.

Program language used is FORTRAN IV, and the computations are carried out by an IBM 370/155 system.

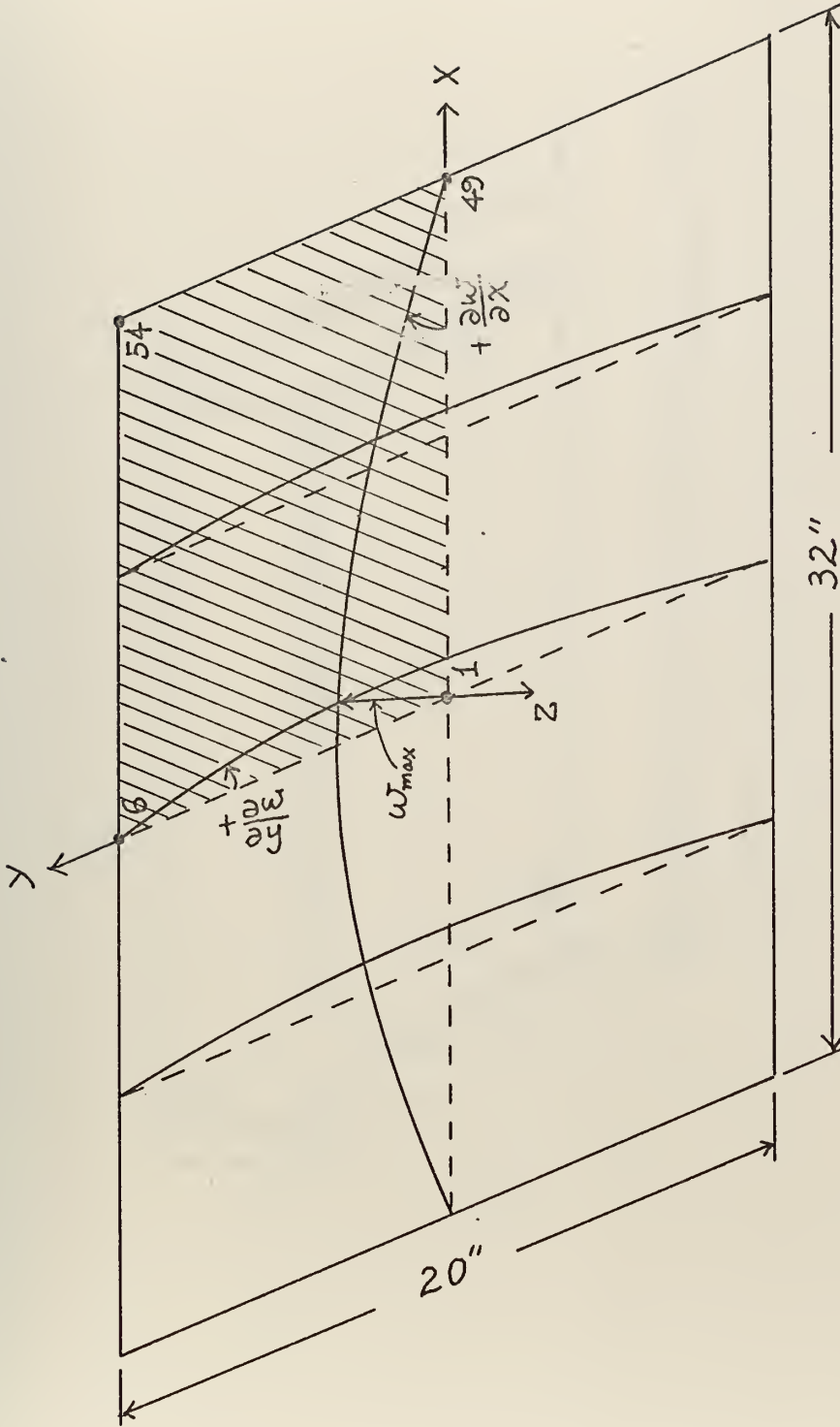


Figure 9 Global Coordinate System and Shape of Welded Deflection

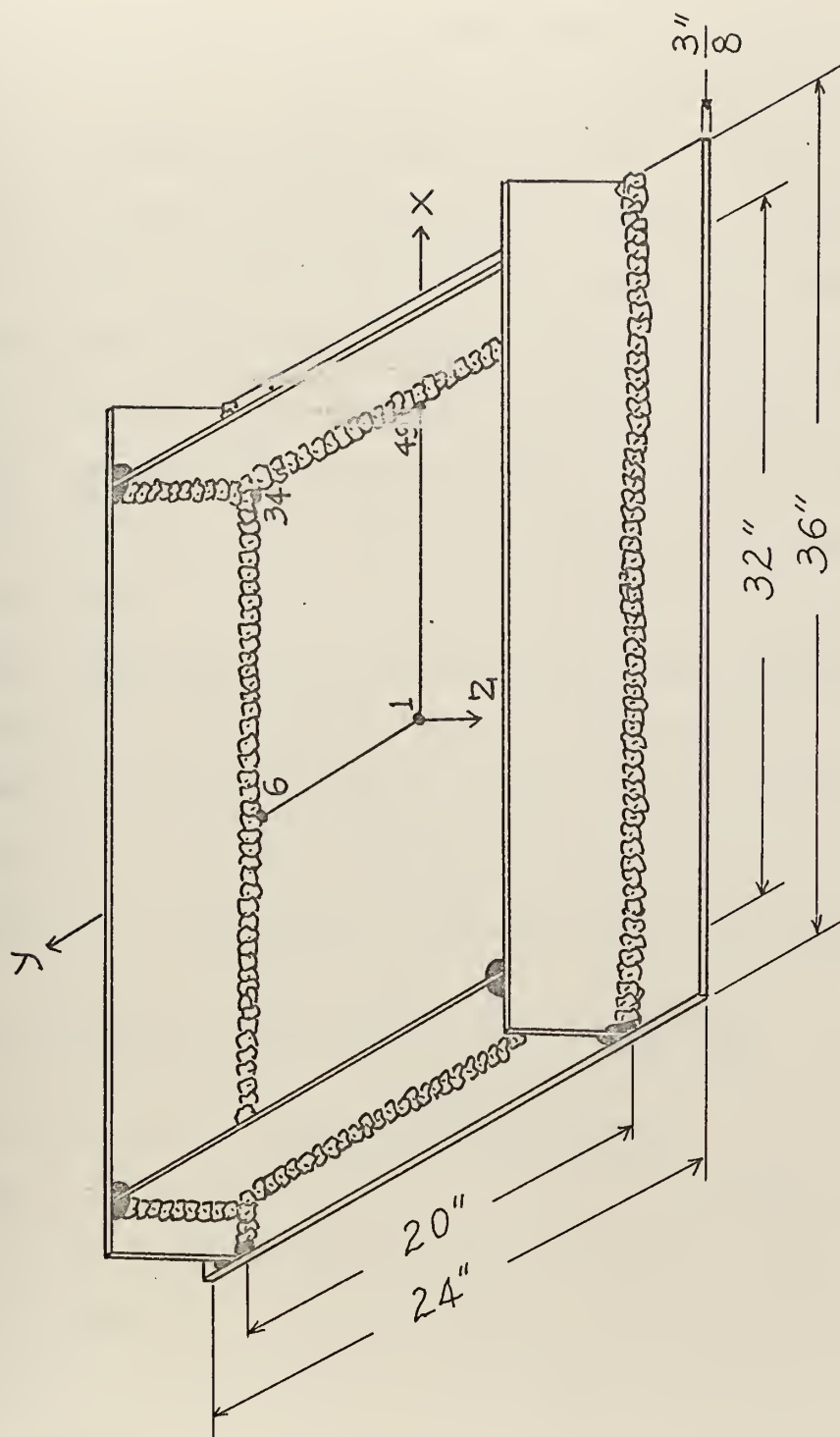


Figure 10 A Panel Structure Analyzed by Experiment and Finite Element Method

III RESULTS

As computer inputs, one steel panel structural dimension, as shown in Figure 10, for which the experiment conducted and weight of consumed welding rods are given, 3 gr/cm., (4,19) are used to predict the deformations, angular changes, and twisting angles at the nodal points. Here, the value of the one dimensional free joint angular change along the welded edge, θ_0 , has been read from the results of Figure 3 and the equivalent constrained welding moment, C, is being evaluated using equation (8). The numerical values of the input data are as follows:

Half-length of the plate in the x- and y-directions, respectively, $L_x = 16$ inches and $L_y = 10$ inches.

Number of finite elements in the x- and y-directions, respectively, $M = 8$ and $N = 5$.

Equivalent constrained welding moment from equation (8)

$$C = \frac{t^4}{1 + W/5} = \frac{(25.4 \times 3/8)^4}{1 + 3/5} = 5200 \frac{\text{kg-mm}}{\text{mm rad.}}$$

where W is given by 3 gr/cm from experiment.

Therefore,

$$C = 1.13 \times 10^4 \text{ lb-in/in rad.}$$

Poisson's ratio:

$$\nu = 0.3$$

Young's modulus:

$$E = 30 \times 10^6 \text{ psi.}$$

Thickness of the plate:

$$t = 3/8 \text{ inch.} \quad (69 \text{ a-h})$$

The results of the analysis using the above data are listed in Appendix E, and these results are plotted in the Figures 11 and 12 to be compared with experimental results. As can be noticed from Figures 11 and 12, the finite element results are lower value than the experiments, and the maximum deflection at the midpoint of the plate which is the most important deflection in reality is only a half of that from the experiment.

To visualize the behavior of the deformations as functions of θ_o and C , series of computations are conducted with different combinations of θ_o and C as shown in Table 1. Yet remaining, the rest of the input data are unchanged, and the maximum deflections at nodal point 1 are plotted in terms of θ_o and C as in Figures 13 and 14, and the maximum angular changes at node 6 and 49 are plotted in the Figures 15 and 16. Only the maximum deflections at node 1, and the maximum angular changes at node 6 and 49 from the results of series computations are tabulated in Tables 2 and 3.

Finally, computed results using $\theta_o = 55 \times 10^{-3}$ and $C = 4.2 \times 10^4$ are listed in Appendix E, and also plotted in Figures 11 and 12 to compare with the experiment. As can be seen from Figures 11 and 12, these results are very close to the experiment and the reason of choosing the above θ_o and C will be discussed in the following section.

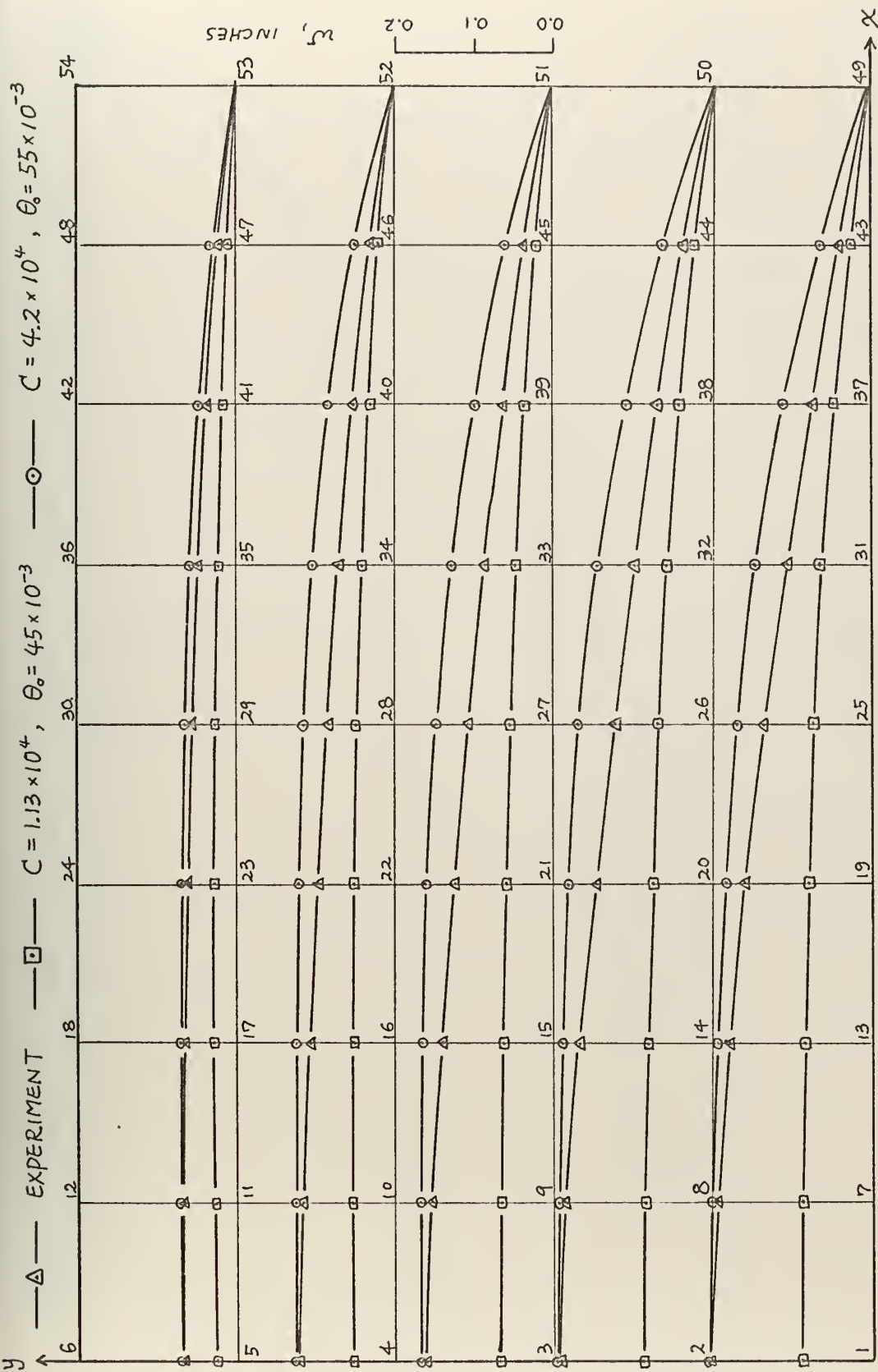


Figure 11 Deflection Comparison with Experiment and Finite Element Analysis

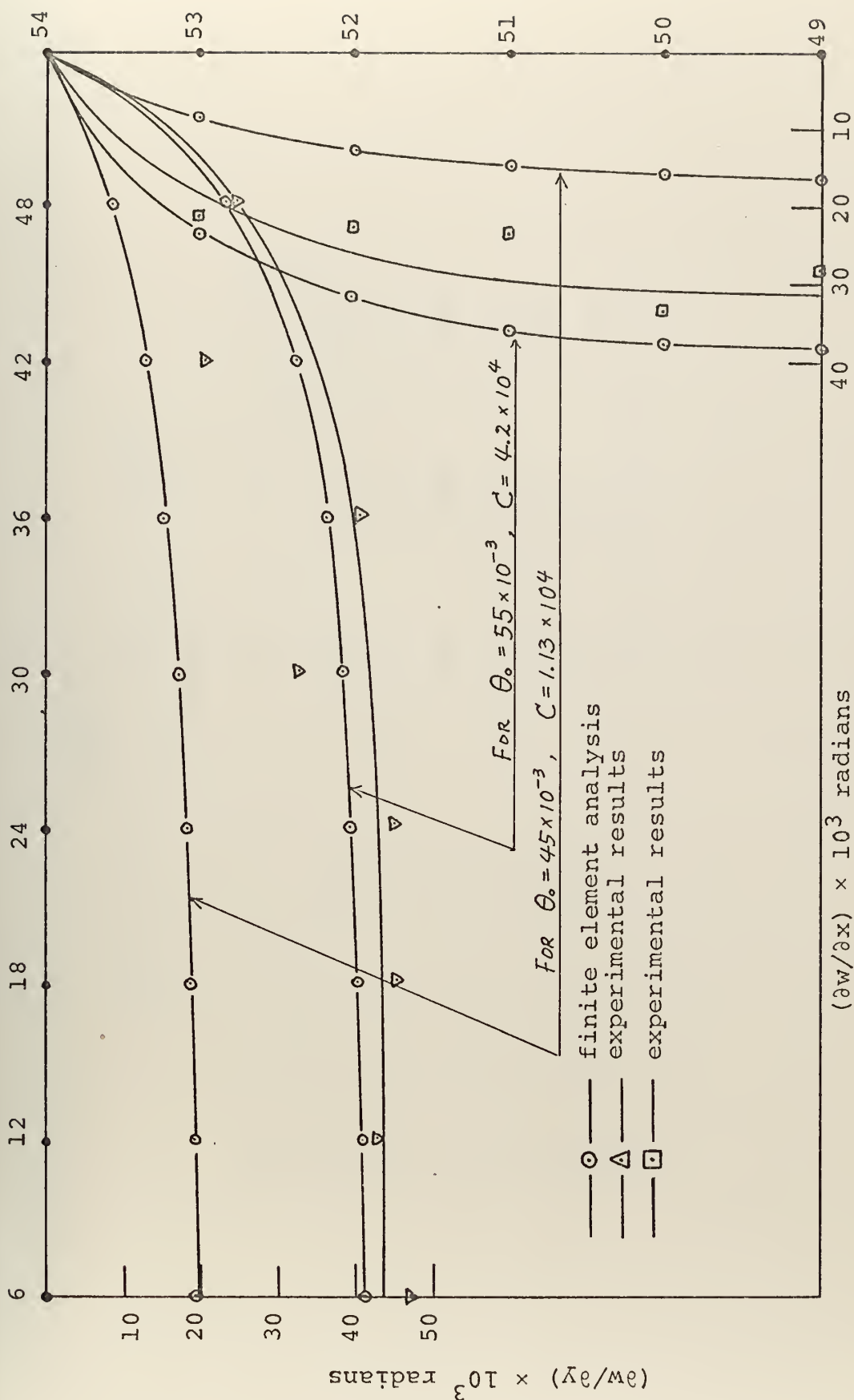


Figure 12 Angular Changes along the Welded Boundary Compared with Experiment and Finite Element Analysis

Table 1

Maximum Deflection at Node 1 in Inches
with Different Combinations of C and θ_0

$\theta_0 \backslash C$	1.13×10^{-4}	4×10^{-4}	8×10^{-4}	11×10^{-4}
45×10^{-3}	0.092	0.166	0.196	0.206
55×10^{-3}	0.112	0.203	0.240	0.252
65×10^{-3}	0.133	0.240	0.283	0.298
75×10^{-3}	0.153	0.277	0.327	0.343
85×10^{-3}	0.174	0.314	0.370	0.389
95×10^{-3}	0.194	0.351	0.414	0.435

Table 2

Maximum Angular Changes ($\partial w / \partial y$) at Node 6
in 10^3 radians with Different Combinations of C and θ_0

$\theta_0 \backslash C$	1.13×10^4	4×10^4	8×10^4	11×10^4
45×10^{-3}	18.86	32.28	38.67	40.33
55×10^{-3}	23.05	40.68	47.26	49.30
65×10^{-3}	27.23	48.07	55.85	58.26
75×10^{-3}	31.43	55.47	64.45	67.23
85×10^{-3}	35.62	62.86	73.04	76.29
95×10^{-3}	39.81	70.26	81.63	85.16

Table 3

Maximum Angular Changes ($\partial w / \partial x$) at Node 49
in 10^3 radians with Different Combinations of C and θ_0

$\theta_0 \backslash C$	1.13×10^4	4×10^4	8×10^4	11×10^4
45×10^{-3}	16.59	31.00	37.18	39.22
55×10^{-3}	20.28	37.90	45.45	47.94
65×10^{-3}	23.97	44.79	53.71	56.66
75×10^{-3}	27.66	51.68	61.97	65.38
85×10^{-3}	31.35	58.57	70.24	74.09
95×10^{-3}	35.04	65.46	78.50	82.81



Figure 13 w_{\max} at Node 1 versus C with Different θ_0 , Dashed Line Represents Experimental Value.

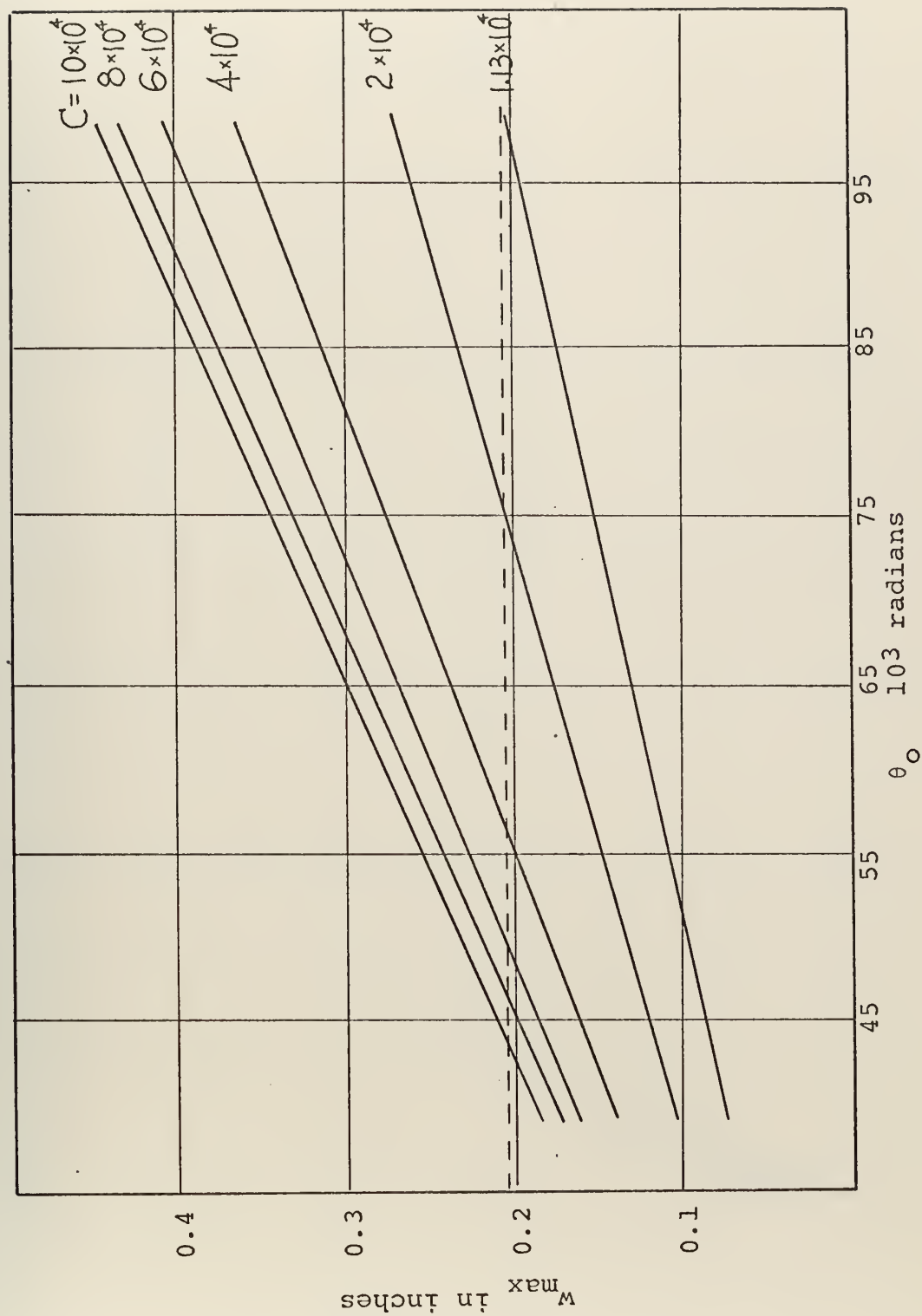


Figure 14 w_{\max} at Node 1 versus θ_0 with Different C, Dashed Line Represents Experimental Value.

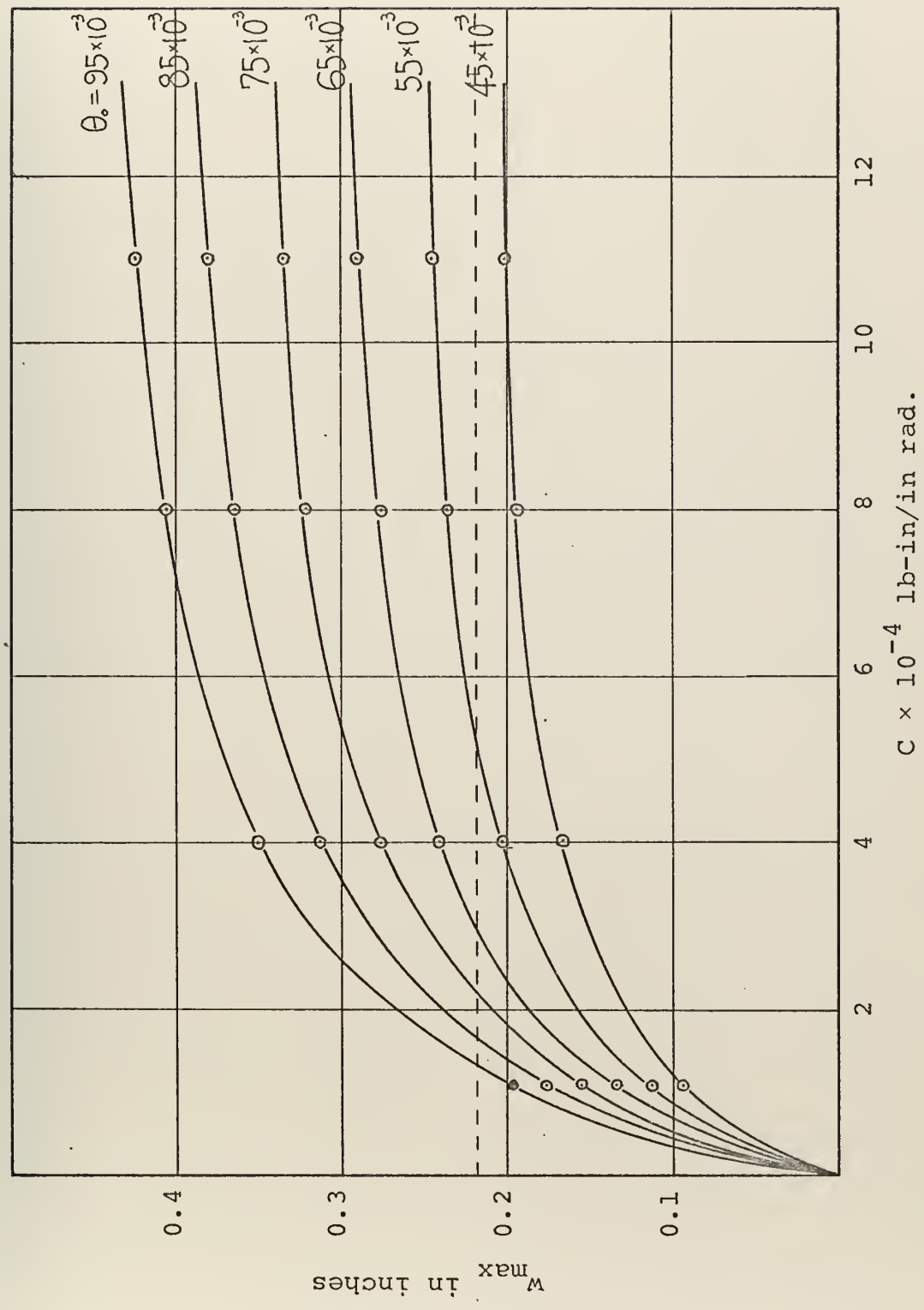


Figure 15 $(\partial w / \partial y)_{\max}$ at Node 6, Dashed Line Represents Experimental Value

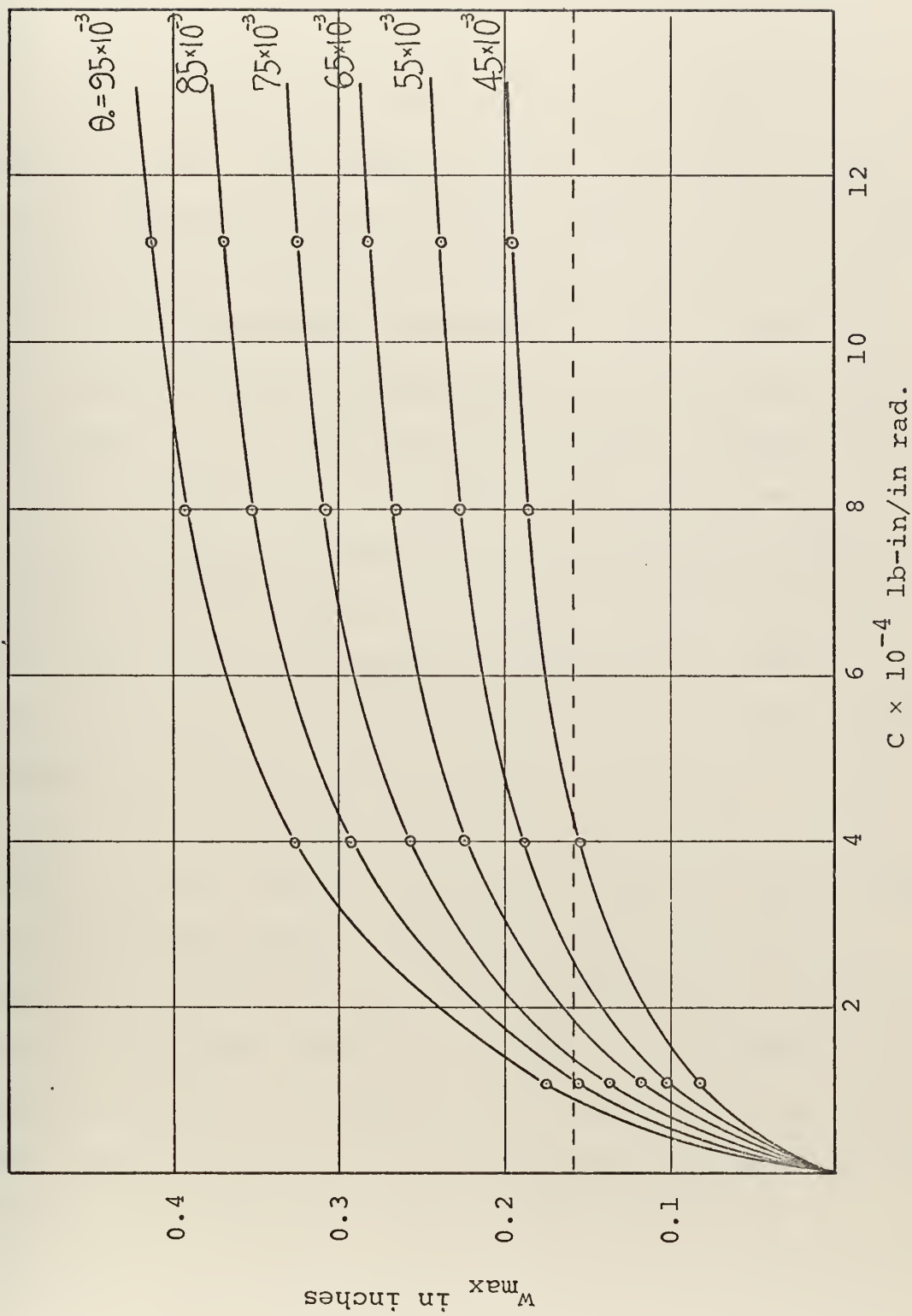


Figure 16 $(\partial w / \partial x)_{\max}$ at Node 49, Dashed Line Represents Experimental Value

IV DISCUSSION OF RESULTS

As has been mentioned in the previous section, the results using $C = 1.13 \times 10^4$ and $\theta_0 = 45 \times 10^{-3}$ are only a half of the experimentally measured values as shown in Figures 11 and 12. The reasons of this difference are not yet known. These may result from the assumption of the elastic behavior of the welded deformation, or from the uncertainty of the experimental measurement itself because the deflections and angular changes which are only of the order of magnitude of 10^{-1} inches and 10^{-2} radians, respectively. Yet, other possibilities are from the value of θ_0 and C .

To investigate the behavior of the deformation with respect to θ_0 and C , series of computations have been conducted. The plotting of maximum deflections at node 1 versus C with changing θ_0 as in Figure 13 shows some insight of deflection behavior with varying C . In here, it is obvious that the deflections are more rapidly changed with changing θ_0 than C changes, which means that the deflections are more strongly dependent function of θ_0 rather than C . Furthermore, the deflection reaches rapidly to an asymptotic value as C increases, which can be seen clearly from the one dimensional deflection, equation (41) where the term $2EI/\ell C$ becomes small compared to 1 as C increases; therefore, in the limit case, the deflection can be expressed only by

$$w = \ell \theta_0 / 4.$$

This behavior of the deflection with changing C are clearly shown in Figure 13. Also from Figure 14, in which the maximum deflections are plotted with respect to θ_0 , it is obvious that the deflections are rapidly increased as θ_0 increases.

If assumed that the difference between experiment and computed results are caused from the possibility of different values of θ_0 and C , then the question is how to choose θ_0 and C , such that the computed maximum deflection becomes the same to that from the experiment. From Figures 12 and 13, the combinations of θ_0 and C to satisfy the given value of maximum deflection from the experiment can be predicted. However, the best combination is not known, but for the known experiment value of maximum deflection it may be safe to choose θ_0 and C such that the changes of both θ_0 and C from the calculated value be minimized. In other words, superposing Figures 13 and 14 as shown in Figure 17, the point which gives the same reading of θ_0 and C in both ways as shown by the small circle, which is read as $\theta_0 = 55 \times 10^{-3}$ and $C = 4.2 \times 10^4$, will be the optimal combination to satisfy the given maximum deflection. The comparison of the results using this θ_0 and C to the experiments, as in Figures 11 and 12, shows that this approach gives reasonably good results.

As has been seen from above, there exists many uncertainties for the two dimensional welding analysis, but the welding deformation phenomena can be predicted with a reasonable degree of accuracy by the finite element analysis.

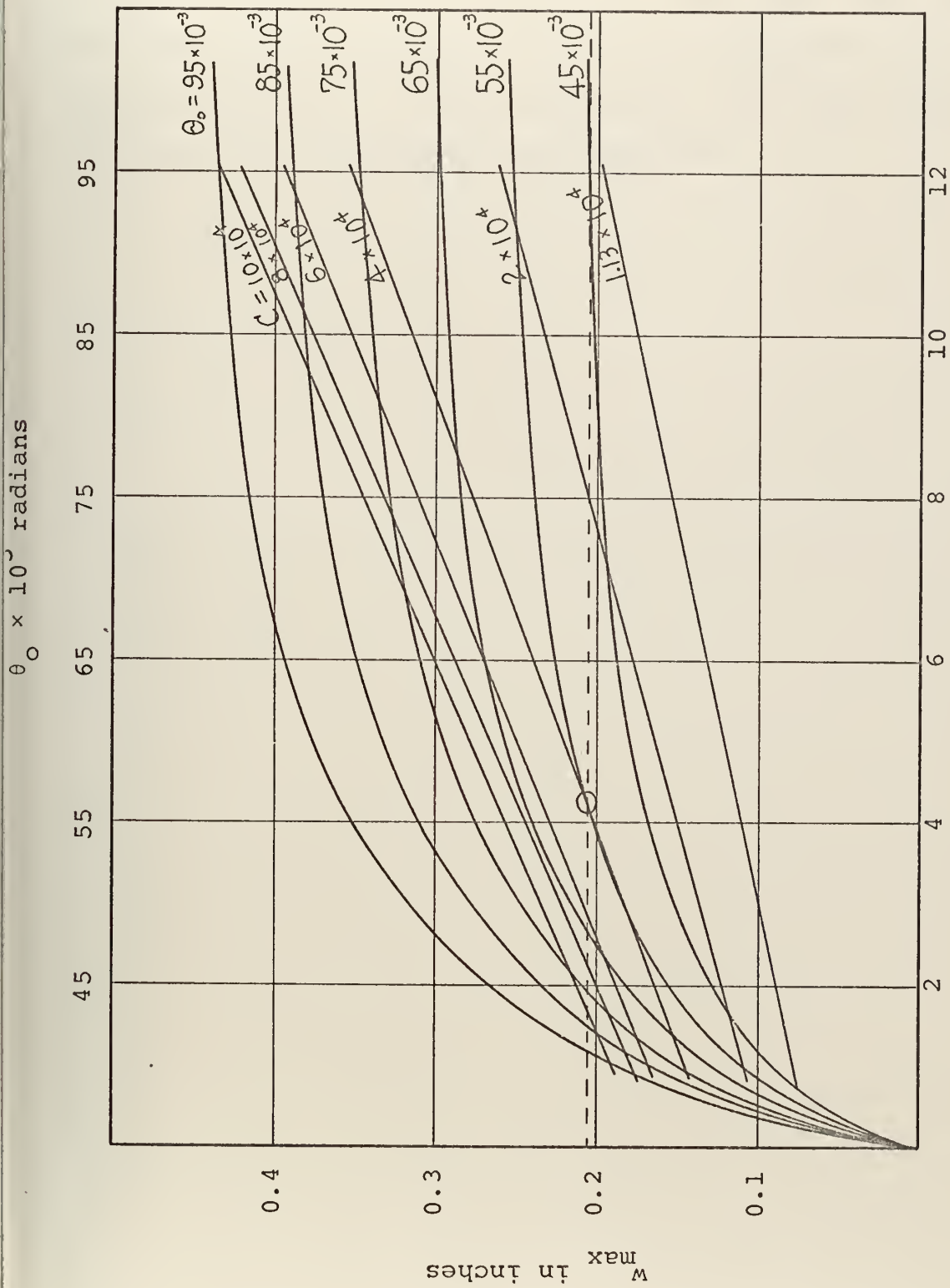


Figure 17 w_{\max} at node 1 after superposition of Figures 13 and 14, $C \times 10^{-4}$ lb-in/in. rad. dashed line represents experimental value.

Furthermore, the empirical formula of C given by equation (8) may not be simply applied to the two dimensional case, and also does the value of θ_0 . If it is so, measuring only the maximum deflection in future experiments, the best combination of θ_0 and C can be predicted as discussed.

V CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

From the discussion of the previous section, it can be concluded as:

1. The finite element analysis for the two dimensional welding deformations which are assumed to be elastic deformation gives reasonable prediction of deflections.
2. The searching technique of optimal combination of θ_0 and C for a given deflection can be used as shown already, if there exists the uncertainty of θ_0 and C.
3. In the future experiments, this analysis gives the order of magnitude feeling of the deflection measured after welding. It is extremely difficult to measure correct deflections in experiments because the order of magnitude to be measured is very small; therefore, the inherent uncertainties always exist.
4. The magnitude of importance of the welding deformation variables such as θ_0 and C are clearly seen, therefore, more care should be concentrated in measuring θ_0 rather than C.
5. Simple empirical formula for C given by equation (8) may not cover the wide range of the values of C; furthermore, as can be derived from equation (41) it may be logical to express C in terms of maximum deflection, free joint angular change, and rigidity,

such as:

$$C = \theta_0 / 4w_{\max} - 2EI/\ell \quad (70)$$

in which θ_0 is a function of a given welding condition and w_{\max} is the constrained maximum deflection at midpoint of the span for the same condition of the welding as of θ_0 . Further, the elastic rigidity and plate dimension terms are expressed by the term of $2EI/\ell$.

B. Recommendations

In this analysis, the assumption used is the elastic behavior of deformations. As stated in the conclusion, it is still believed reasonable to use the elastic theory, but for more accurate prediction, local plastic deformation phenomena should be included in the analysis.

If the elastic assumption is to be used, more careful analysis to evaluate the value of θ_0 and C are recommended in future research. Also, a more rational approach to express C is recommended.

In future research, it is recommended that series of experiments be conducted to measure the maximum plate deformation only, and express C and θ_0 in terms of welding and plate dimension. In this way, the computer inputs are always available for any prediction of deformation for the two dimensional panel structures.

APPENDIX A

Definitions of Matrices

$$\underset{\sim}{C} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \\ c_{13} \\ c_{14} \\ c_{15} \\ c_{16} \end{pmatrix} \quad \underset{\sim}{q} = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \\ q_9 \\ q_{10} \\ q_{11} \\ q_{12} \\ q_{13} \\ q_{14} \\ q_{15} \\ q_{16} \end{pmatrix} = \begin{pmatrix} w_1 \\ w_{x_1} \\ w_{y_1} \\ w_{xy_1} \\ w_2 \\ w_{x_2} \\ w_{y_2} \\ w_{xy_2} \\ w_3 \\ w_{x_3} \\ w_{y_3} \\ w_{xy_3} \\ w_4 \\ w_{x_4} \\ w_{y_4} \\ w_{xy_4} \end{pmatrix}$$

$$\tilde{H} = \begin{pmatrix} 0 & 0 & 0 & 0 & 2 & 0 & 6x & 2y & 0 & 0 & 6xy & 2y^2 & 0 & 6xy^2 & 2y^3 & 6xy^3 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2x & 6y & 0 & 2x^2 & 6xy & 2x^3 & 6x^2y & 6x^3y \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 4x & 4y & 0 & 6x^2 & 8xy & 6y^2 & 12x^2y & 12xy^2 & 18x^2y^2 \end{pmatrix}$$

$$\tilde{F}_x = [0 \ 0 \ 0 \ 1 \ x \ 0 \ 2y \ 0 \ x^2 \ 2xy \ 3y^2 \ x^3 \ 2x^2y \ 3xy^2 \ 2x^3y \ 3x^2y^2 \ 3x^3y^2]$$

$$\tilde{F}_y = [0 \ 1 \ 0 \ 0 \ y \ 2x \ 0 \ 3x^2 \ 2xy \ y^2 \ 0 \ 3x^2y \ 2xy^2 \ y^3 \ 3x^2y^2 \ 2xy^3 \ 3x^2y^3]$$

APPENDIX B

Example of Assembling

As an example, the assembling procedure for a quarter part of the plate divided by two finite element shown in Figure 18a will be discussed.

For the element 1, as shown in Figure 18b, the stiffness matrix in a local coordinate can be written from equation (58a, b),

$$\underline{k}_{t_1} = \underline{k} + \underline{k}_{w_x} \quad (A-1)$$

where \underline{k}_{t_1} is 16 by 16 symmetric matrix. And from equation (58d), the load matrix for the element 1 is,

$$\underline{Q}_{t_1} = \underline{Q}_x \quad (A-2)$$

where \underline{Q}_{t_1} is 16 by 1 matrix. By the same way for element 2, the stiffness and load matrix can be written using equation (58a, b, c) and (58d, e), respectively,

$$\underline{k}_{t_2} = \underline{k} + \underline{k}_{w_x} + \underline{k}_{w_y} \quad (A-3)$$

$$\underline{Q}_{t_2} = \underline{Q}_x + \underline{Q}_y \quad (A-4)$$

where \underline{k}_{t_2} is 16 by 16 symmetric matrix and \underline{Q}_{t_2} is 16 by 1 matrix.

At every nodal point there are four unknowns (w, w_x, w_y, w_{xy}), therefore, after assembling element 1 and 2, the equations have to be solved with proper boundary and symmetry conditions are 24 by 24 simultaneous equations, which is in the matrix form:

$$\delta \underline{\underline{q}}^T [\underline{\underline{K}} \underline{\underline{q}} - \underline{\underline{Q}}] = 0 \quad (\text{A-5})$$

where the matrix $\underline{\underline{K}}$ is 24 by 24 symmetric matrix, $\underline{\underline{Q}}$ is 24 by 1, $\underline{\underline{q}}$ is 24 by 1, and $\underline{\underline{q}}^T$ is 1 by 24 matrix.

All the quantities in equation (A-5) are defined by the global coordinate, therefore, the assembling procedure is to locate the quantities defined by equation (A-1, A-2, A-3, A-4) to the proper position in equation (A-5). This operation can easily be established comparing each nodal points numbered in local coordinates to that of global coordinates:

Element 1, nodal points

Element 2, nodal points

Coordinates

Coordinates

local	global	local	global
1	1	1	3
2	3	2	5
3	4	3	6
4	2	4	4

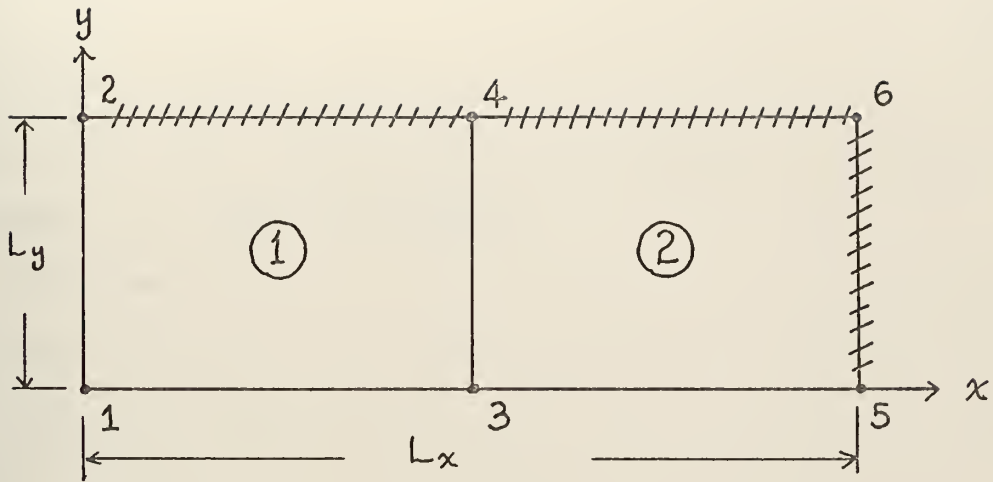
And noticing that numbering of the generalized displacement, q_i , follows the relation,

$$i = 4m - g \quad (\text{A-6})$$

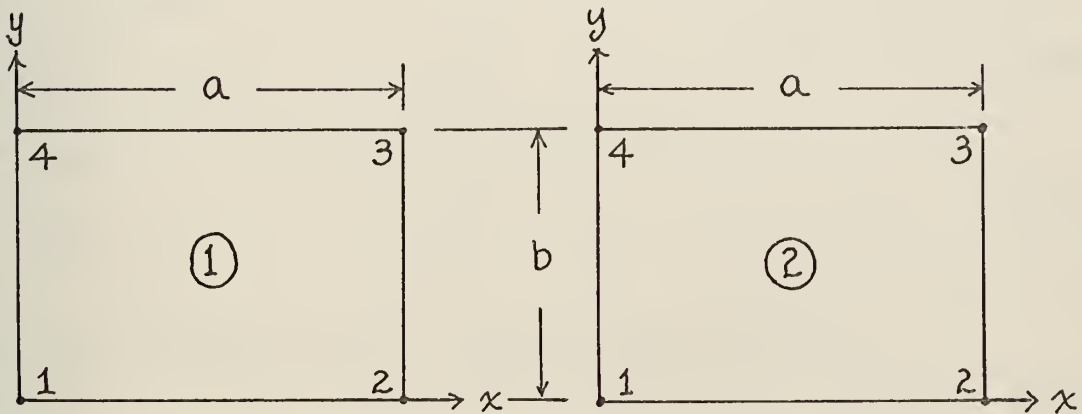
$$j = 4n - h \quad (\text{A-7})$$

where i and j represent the index for generalized displacements, m and n for nodal numbers, g and h are set to be 3 for w_x , 2 for w_y , 1 for w_x , and 0 for w_{xy} .

For example, one of the stiffness matrix elements at nodal point 4 in global coordinate can be calculated.



a. Nodal points numbering in global coordinate



b. Nodal points numbering in local coordinate

Figure 18 Elements and Nodal Points Numbering in Global and Local Coordinate for Sample Assembling

From equation (A-6) and (A-7),

$$i = 16 - g \quad (A-8)$$

$$j = 16 - h \quad (A-9)$$

But the same node 4 becomes node 3 at element 1 and node 4 at element 2 in local coordinate.

For element 1,

$$(i)_1 = 12 - g \quad (A-10)$$

$$(j)_1 = 12 - h \quad (A-11)$$

and for element 2,

$$(i)_2 = 16 - g \quad (A-12)$$

$$(j)_2 = 16 - h \quad (A-13)$$

Therefore, if $g = 1$ and $h = 2$, then from equation (A-8),

(A-9), and (A-5),

$$K_{ij} = K_{15,14} \quad (A-14)$$

From equations (A-10), (A-11), and (A-1)

$$k_{t_1} \quad 11,10 \quad (A-15)$$

and from equations (A-12), (A-13), and (A-3),

$$k_{t_2} \quad 15,14 \quad (A-16)$$

Therefore, element of the stiffness matrix in global coordinate becomes,

$$K_{15,14} = k_{t_1} \quad 11,10 + k_{t_2} \quad 15,14 \quad (A-17)$$

All other stiffness matrix element can be expressed by the same manner described above, and the programs of assembling the stiffness matrix are available in FEABL. (13)

APPENDIX C

Description of Input Data

Input data for the programs listed in Appendix D are:

C_x , C_y , θ_o , D , t , E , L_x , L_y , M and N as described in section III. In addition to these, control card for calculation of different cases in each job with changing above input variables are considered. By using this calculation control input card in each job, series of calculation with changing some of the input variables can be calculated, and the deformation behavior with respect to these changing variables can easily be visualized by plotting the deformation versus these variables.

Detailed input data card arrangement and format are listed in Table 4.

Table 4

Input Data Layout Form

<u>Card Column</u>	<u>Input Format</u>	<u>Program Symbol</u>	<u>Definition and Unit</u>	<u>Sample Input</u>
<u>Card Number 1</u>				
1-5	I5	NCASE	Number of cases involved in each job	14
<u>Card Number 2</u>				
1-15	E15.7	LX	Length of 1/4 plate in x, inches	16.0E+00
16-30	E15.7	LY	Length of 1/4 plate in y	10.0E+00
31-35	I5	M	Number of elements in x	8
36-40	I5	N	Number of elements in y	5
<u>Card Number 3</u>				
1-12	E12.7	CX	Equivalent constrained welding moment as in Figure 6, lb-in/in rad.	4.20E+04
13-24	E12.7	CY	Equivalent constrained welding moment as in Figure 6, lb-in/in rad.	4.20E+04
25-36	E12.7	THETAZ	Free joint angular change, θ_0 , radians	55.0E-03
37-48	E12.7	GNU	Poisson's ratio, D	0.30E+00
49-60	E12.7	THK	Plate thickness, t, inches	0.375E+00
61-72	E12.7	E	Young's modulus, E, lb/in. ²	30.0E+06

APPENDIX D

Listing of Programs


```

C DAWPS PROGRAM
C DISTORTION ANALYSIS OF WELDED PANEL STRUCTURE
C FORMULATED BY D. SHIN
C PROGRAMMED BY SUSAN E. FRENCH.....ASPL AT MIT FER. 1972
C THIS PROGRAM USES FEARL, FINITE ELEMENT ANALYSIS BASIC
C LIBRARY, WRITTEN BY DR. OSCAR ORRINGER OF THE AERONAUTICS AND
C ASTRONAUTICS DEPT. OF MIT
C REAL*4 LX,LY
C DIMENSION REAL(8000),INTGR(8000)
C DIMENSION D(3,3),G(16,16),ELK1(16,16),ELK2(16,16),ELK3(16,16),
C     ELK4(16,16),Q1(16),Q2(16),Q3(16),Q4(16)
C COMMON /IO/ KREAD,KWRITE,KPUNCH
C COMMON /SIZE/ NET, NDT
C COMMON /BEGIN/ ICON,IKOUNT,ILNZ,IMASTR,IQ,IK
C COMMON /END/ LCON,LKOUNT,LLNZ,LMASTR,LQ,LK
C COMMON /CONST/ M,N,A,B,CX,CY,BETA,THETAZ,ABBET,HALFA,HALFB,ASQ,ACU
C     IB,BSQ,BCUB,CXB2,CYA2,BTHCX,ATHCY
C COMMON /GAUSS/ WRAR(3),XBAR(3),YBAR(3)
C COMMON /MESH/ NDE,NNT,LTYPE(200)
C EQUIVALENCE (REAL(1), INTGR(1))
C DEFINE LDIM=DIMENSION OF PEAL AND INTGR
C LDIM=8000
C DEFINE PEAD , WRITE AND PUNCH DATA SET REFERENCE NUMBERS
C KREAD=5
C KWRITE=6
C KPUNCH=7
C READ ALL INPUT DATA
C READ(KREAD,900) NCASE
C     FORMAT(I5)
C     DO 100 NRUN=1,NCASE
C     READ (KREAD,1000) LX,LY,M,N,CX,CY,THETAZ,GNU,THK,E
C     FORMAT(2E15.7,2I5/6E12.7)
C     LX=LENGTH IN X DIRECTION, LY=LENGTH IN Y DIRECTION, M= NO. MESHES IN X
C     DIRECTION, N=NO. OF MESHES IN Y DIRECTION,CX AND CY ARE CONSTANTS,
C     THETAZ=THETA SUB ZERO=AN ANGLE IN RADIANS, GNU=NU=POISSONS RATIO,

```



```

C      THK=SMALL T=THICKNESS, E= YOUNGS MODULUS
C
C      NET=NO. OF ELEMENTS (TOTAL) IN MESH,NDE=NO. OF DEGREES OF FREEDOM
C      PER MESH ELEMENT,NNT=NO. OF NODES (TOTAL) IN MESH,NDT=NO. OF
C      DEGREES OF FREEDOM (TOTAL) IN MESH,NCON= NO. OF CONSTRAINTS
C      (BOUNDARY CONDITIONS) APPLIED TO TOTAL MESH
C      NET=N*M
C      NDE=16
C      NNT=(N+1)*(M+1)
C      NDT=4*NNT
C      MASTRL=NET*(1+NDE)
C      NCON=4*(N+M+1)
C      CALCULATE SOME CONSTANTS
C      BETA=E*THK**3/(12.*(1.-GNU*GNU))
C      A=LX/FLOAT(M)
C      B=LY/FLOAT(N)
C      HALFA=.5*A
C      HALFB=.5*B
C      ASQ=A*A
C      ACUB=ASQ*A
C      BSQ=B*B
C      BCUB=BSQ*B
C      ABBET=A*B*BETA/4.
C      CXB2=.50*CX*B
C      CYA2=.50*CY*A
C      BTHCX=.5*B*THETAZ*CX
C      ATHCY=.5*A*THETAZ*CY
C      CALCULATE T MATRIX, D MATRIX AND (INVERSE OF T) = G MATRIX
C      CALL TDGMAT(GNU,D,G)
C      CALCULATE DATA IN BEGIN AND END LABELLED COMMON
C      CALL SETUP(LDIM,NCON,MASTRL,REAL,INTGR)
C      CALCULATE AND STORE MASTER VECTOR DATA RELATING ELEMENTS AND
C      THE DEGREE OF FREEDOM NUMBERS (GLOBAL) AT EACH CORNER
C      CALL NODDOF(REAL,INTGR)
C      FILL IN KOUNT AND LNZE SECTIONS OF INTGR AND CALCULATE LK
C      CALL ORK(LDIM,REAL,INTGR)

```


C DEFINE CONSTANTS NEEDED FOR GAUSSIAN QUADRATURE INTEGRATION

```

WBAR(1)=.5555556
WBAR(2)=.8888889
WBAR(3)=.5555556
XBAR(1)=-.7745967
XBAR(2)=0.
XBAR(3)=+.7745967
YBAR(1)=-.7745967
YBAR(2)=0.
YBAR(3)=+.7745967

```

C CALCULATE ELEMENT STIFFNESS MATRIX FOR NO WELD

```

CALL STIF1(D,G,ELK1,Q1)
CALL STIF2(G,ELK2,Q2)
CALL STIFX(G,ELK3,Q3)
DO 40 KK=1,NDE
  Q2(KK)=Q2(KK)+Q1(KK)
  Q4(KK)=Q2(KK)+Q3(KK)
  Q3(KK)=Q3(KK)+Q1(KK)
DO 30 LL=1,NDE
  ELK2(KK,LL)=ELK1(KK,LL)+ELK2(KK,LL)
  ELK4(KK,LL)=ELK2(KK,LL)+ELK3(KK,LL)
  ELK3(KK,LL)=ELK3(KK,LL)+ELK1(KK,LL)

```

30 CONTINUE
40 CONTINUE

, C ASSEMBLE TOTAL STIFFNESS MATRIX AND TOTAL Q VECTOR

```

DO 60 J=1,4
DO 50 LNUM=1,NET
  IF(LTYPE(LNUM).NE.J) GO TO 50
  GO TO (42,44,46,48), J
42 CALL ASEFMBL(LNUM,NDE,ELK1,Q1,REAL,INTGR)
   GO TO 50
44 CALL ASEFMBL(LNUM,NDE,ELK2,Q2,REAL,INTGR)
   GO TO 50
46 CALL ASEFMBL(LNUM,NDE,ELK3,Q3,REAL,INTGR)
   GO TO 50
48 CALL ASEFMBL(LNUM,NDE,ELK4,Q4,REAL,INTGR)

```



```

50 CONTINUE
60 CONTINUE
C   FILL IN CONSTRAINT VECTOR FOR ENTIRE STRUCTURE
CALL HOLD(N,M,ICON,NCON,IQ,INTGR,REAL)
C   APPLY CONSTRAINTS TO ASSEMBLED K MATRIX
CALL BCON(REAL,INTGR)
CALL FACTSD(REAL,INTGR)
CALL SIMULQ(ENERGY,REAL,INTGR)
C   PRINT ALL INPUT DATA
WRITE(KWRITE,2000) LX,LY,M,N,CX,CY,THETAZ,GNU,THK,E
2000 FORMAT(1H1/1H0/'ODISTORTION ANALYSIS OF WELDED PANEL STRUCTURE.....
1BY D. SHIN'/4H0LX=,E12.5,5H, LY=,E12.5,23H, M=NO. ELEMENTS X DIR=,
213/20H0NO. ELEMENTS Y DIR=,13,5H, CX=,E12.5,5H, CY=,E12.5/9H0THETA
3 0=,E12.5,25H RADIANS, POISSONS RATIO=,E12.5/11H0THICKNESS=,E12.5,
417H, YOUNGS MODULUS=,E12.5)
WRITE(KWRITE,2120)
2120 FORMAT(5H0NODE,7X,1HW,12X,2HDW,12X,2HDX,8X,10+D( DW/DY )/1H+,23X,4
1H-----,10X,4H-----,6X,12H-----/1H ,24X,2HDX,12X,2HDY,12X,2HDX
2)
C   ISTART=IQ-4
DO 80 NO=1,NNT
ISTART=ISTART+4
IEND=ISTART+3
WRITE (KWRITE,2130) NO,(REAL(J),J=ISTART,IEND)
2130 FORMAT(14,4E14.5)
80 CONTINUE
100 CONTINUE
STOP
END

```



```

SUBROUTINE TDGMAT(GNU,D,G)
CALCULATE T, D AND G=INVERSE OF D MATRICES
COMMON /CONST/ M,N,A,B,CX,CY,BETA,THETA,Z,ABSET,HALFA,HALFR,ASQ,ACU
1B,BSQ,BCJB,CX82,CYA2,BTHCX,ATHCY
COMMON /IO/ KREAD,KWRITE,KPUNCH
DIMENSION D(3,3),G(16,16),T(16,16),LL(16),MM(16),ALIN(256)
CALL ERASE (T,256)
T(1,1)=1.
T(2,2)=1.
T(3,3)=1.
T(4,4)=1.
T(5,1)=1.
T(5,2)=A
T(5,5)=ASQ
T(5,7)=ACUB
T(6,2)=1.
T(6,5)=2.*A
T(6,7)=3.*ASQ
T(7,3)=1.
T(7,4)=A
T(7,8)=ASQ
T(7,11)=ACUB
T(8,4)=1.
T(8,8)=T(6,5)
T(8,11)=T(6,7)
T(9,1)=1.
T(9,2)=A
T(9,3)=B
T(9,4)=B*A
T(9,5)=ASQ
T(9,6)=BSQ
T(9,7)=ACUB
T(9,8)=ASQ*B
T(9,9)=A*BSQ
T(9,10)=BCUB
T(9,11)=ACUB*B

```


$T(9,12)=ASQ*BSQ$
 $T(9,13)=A*BCUB$
 $T(9,14)=ACUB*BSQ$
 $T(9,15)=ASQ*BCUB$
 $T(9,16)=ACUB*BCUB$
 $T(10,2)=1.$
 $T(10,4)=B$
 $T(10,5)=T(6,5)$
 $T(10,7)=T(8,11)$
 $T(10,8)=2.*A*B$
 $T(10,9)=BSQ$
 $T(10,11)=3.*T(9,8)$
 $T(10,12)=2.*T(9,9)$
 $T(10,13)=BCUB$
 $T(10,14)=3.*T(9,12)$
 $T(10,15)=2.*T(9,13)$
 $T(10,16)=3.*T(9,15)$
 $T(11,3)=1.$
 $T(11,4)=A$
 $T(11,6)=2.*B$
 $T(11,8)=ASQ$
 $T(11,9)=T(10,8)$
 $T(11,10)=3.*BSQ$
 $T(11,11)=ACUB$
 $T(11,12)=2.*T(9,8)$
 $T(11,13)=3.*T(9,9)$
 $T(11,14)=2.*T(9,11)$
 $T(11,15)=T(10,14)$
 $T(11,16)=3.*T(9,14)$
 $T(12,4)=1.$
 $T(12,8)=T(10,5)$
 $T(12,9)=T(11,6)$
 $T(12,11)=T(10,7)$
 $T(12,12)=4.*T(9,4)$
 $T(12,13)=T(11,10)$
 $T(12,14)=6.*T(9,8)$


```

T(12,15)=6.*T(9,9)
T(12,16)=9.*T(9,12)
T(13,1)=1.
T(13,3)=B
T(13,6)=BSQ
T(13,10)=BCU9
T(14,2)=1.
T(14,4)=E
T(14,9)=BSQ
T(14,13)=BCUB
T(15,3)=1.
T(15,6)=T(11,6)
T(15,10)=T(11,10)
T(16,4)=1.
T(16,9)=T(15,6)
T(16,13)=T(15,10)
D(1,1)=1.
D(2,1)=GNU
D(3,1)=0.
D(1,2)=GNU
D(2,2)=1.
D(3,2)=0.
D(1,3)=0.
D(2,3)=0.
D(3,3)=.5*(1.-GNU)
C CONVERT T MATRIX TO A VECTOR FOR INPUT TO SUBROUTINE MINV OF SSP
L=0
DO 30 J=1,16
DO 30 I=1,16
L=L+1
30 ALIN(L)=T(I,J)
C INVERT MATRIX T
C CALL MINV (ALIN,16,DET,LL,MM)
C CHECK THAT DET=DETERMINANT IS NON-ZERO
C IF (DET.NE.0) GO TO 40
C WRITE(KWRITE,3020)

```



```
3020 FORMAT(68H0T MATRIX IS SINGULAR. EXECUTION IS TERMINATED IN SUBROU  
1TIME TDGMAT./1H1)  
      STOP  
C      CONVERT ALIN TO A 16X16 MATRIX=THE G MATRIX  
40      L=0  
      DO 50 J=1,16  
      DO 50 I=1,16  
      L=L+1  
50      G(I,J)=ALIN(L)  
      RETURN  
      END
```



```

SUBROUTINE HOLD (N,M,ICON,NCON,IQ,INTGR,REAL)
THIS SUBROUTINE GENERATES THE BOUNDARY CONDITIONS ASSUMING
THAT ONLY THE UPPER RIGHT CORNER OF A RECTANGULAR PANEL IS BEING
CONSIDERED. THE ENTIRE PANEL IS ASSUMED TO BE WELDED AROUND THE EDGES
DIMENSION INTGR(2),REAL(2)
JAY=ICON-1
JDOF1=-3
MP1=M+1
NP1=N+1
DO 120 JJ=1,MP1
DO 100 II=1,NP1
DEFINE THE GLOBAL DOF NDS. FOR THE AT Y(II),X(JJ)
JDOF1=JDOF1+4
JDOF2=JDOF1+1
JDOF3=JDOF2+1
JDOF4=JDOF3+1
TEST TO SEE IF (X,Y) IS AN EDGE POINT
IF(JJ.NE.4P1.AND.JJ.NE.1.AND.II.NE.NP1.AND.II.NE.1) GO TO 100
IS AN EDGE POINT
IF(JJ.EQ.1) GO TO 10
GO TO 40
LEFT EDGE
JAY=JAY+1
INTGR(JAY)=JDOF2
JAY=JAY+1
INTGR(JAY)=JDOF4
IF(II.EQ.1) GO TO 20
IF(JI.EQ.NP1) GO TO 30
GO TO 100
LOWER LEFT CORNER
JAY=JAY+1
INTGR(JAY)=JDOF3
GO TO 100
UPPER LEFT CORNER
JAY=JAY+1
INTGR(JAY)=JDOF1

```



```

C      40      GO TO 100
C      NOT ON LEFT EDGE
C      IF(JJ.EQ.NP1) GO TO 70
C      NOT ON RIGHT EDGE OR LEFT EDGE
C      IF(II.EQ.1) GO TO 50
C      IF(II.EQ.NP1) GO TO 60
C      GO TO 100
C      ON LOWER EDGE
C      50      JAY=JAY+1
C      INTEG(JAY)=JDDF3
C      JAY=JAY+1
C      INTEG(JAY)=JDDF4
C      GO TO 100
C      ON TOP EDGE
C      60      JAY=JAY+1
C      INTEG(JAY)=JDDF1
C      JAY=JAY+1
C      INTEG(JAY)=JDDF2
C      GO TO 100
C      ON RIGHT EDGE
C      70      JAY=JAY+1
C      INTEG(JAY)=JDDF1
C      JAY=JAY+1
C      INTEG(JAY)=JDDF3
C      IF(II.EQ.1) GO TO 80
C      IF(II.EQ.NP1) GO TO 90
C      GO TO 100
C      ON LOWER RIGHT CORNER
C      80      JAY=JAY+1
C      INTEG(JAY)=JDDF4
C      GO TO 100
C      ON UPPER RIGHT CORNER
C      90      JAY=JAY+1
C      INTEG(JAY)=JDDF2
C      120 CONTINUE
C      120 CONTINUE

```



```
NCN=JAY
DO 130 I=1,NCN
  KSUB=ICCN-1+I
  *SUB=INTGP(KSUB)
  LSUB=ID-1+MSUB
  REAL(LSUB)=0.
130 CONTINUE
RETURN
END
```

130


```

SUBROUTINE NODDEF(REAL,INTGR)
DIMENSION REAL(2),INTGR(2)
COMMON /SIZE/ NET, NPT
COMMON /BEGIN/ ICON,IKOUNT,ILNZ,IMASTR,IO,IK
COMMON /END/ LCON,LKOUNT,ILNZ,LMASTR,LQ,LK
COMMON /CONST/ M,N,A,B,CX,CY,BETA,THETAZ,ARBET,HALFA,HALFB,ASQ,ACU
1P,BSQ,BCJ3,CXR2,CYA2,BTHCX,ATHCY
COMMON /MESH/ NDE,NNT,LTYPE(200)
C THIS SUBROUTINE CALCULATES THE GLOBAL DOF NUMBERS
KSUB=IMASTR-1
N4P1=4*(N+1)
KPUT=IMASTR+NET
DO 30 IJ=1,M
DO 30 II=1,N
LJUM=II+(JJ-1)*N
INTERNAL (NO EDGE WELDED)
IF (II.LT.N.AND.JJ.LT.M) LTYPE(LNUM)=1
C ALONG RIGHT EDGE (ONLY RIGHT EDGE WELDED)
IF (II.NE.N.AND.JJ.EQ.M) LTYPE(LNUM)=2
C ALONG TOP EDGE (ONLY TOP EDGE WELDED)
IF (II.EQ.N.AND.JJ.LT.M) LTYPE(LNUM)=3
C UPPER RIGHT CORNER (TWO EDGES WELDED)
IF (II.EQ.N.AND.JJ.EQ.M) LTYPE(LNUM)=4
KSUB=KSUB+1
INTGR(KSUB)=KPUT
KDOF=4*(LJUM+JJ-1)-3
DO 20 KK=1,4
C LOWER LEFT CORNER OF MESH
K=KK-1
KSUB1=KPUT+K
INTGR(KSUB1)=KDOF+K
C LOWER RIGHT CORNER OF MESH
KSUB2=KSUB1
KSUB1=KSUB1+4
INTGR(KSUB1)=INTGR(KSUB2)+N4P1
C UPPER RIGHT CORNER OF MESH

```



```

C
KSUP2=KSJ31
KSUB1=KSJ31+4
INTGR(KSJ31)=INTGR(KSUB2)+4
UPPEF LEFT CORNER OF MESH
KSUB1=KSUB1++
KSUP2=KSJ31-4
INTGR(KSJ31)=INTGR(KSUB2)+4
20 CONTINUE
KPUT=KPUT+16
30 CONTINUE
40 CONTINUE
LAST=KSUB1
IF(LMASTER.GT.LAST) GO TO 50
RETURN
50 KSUB1=LAST+1
DO 60 I=KSUB1,LMASTER
60 INTGR(I)=0
RETURN
END

```



```

SUBROUTINE STIFF1(O,G,STIFFM,Q)
THIS SUBROUTINE CALCULATES THE ELEMENT STIFFNESS MATRIX FOR NO WELD
DIMENSION H(3,16),P(16,16),HTOH(16,16),STIFFM(16,16),Q(16),WF(16)
DIMENSION D(3,3),S(15,16)
COMMON /IO/ KREAD,KWRITE,KPUNCH
COMMON /CONST/ 4,N,A,R,CX,CY,BETA,THETAZ,ARBET,HALFA,HALFB,ASQ,ACU
13,RSQ,PCUR,CX32,CY32,RTHCX,ATHCY
COMMON /GAUSS/ WBAR(3),XBAR(3),YBAR(3)
CALL ERASE (4,43,STIFFM,256,P,256,Q,16)
H(1,5)=2.
H(2,6)=2.
H(3,4)=2.
DO 48 I=1,3
XSI=XBAR(I)
XI=WBAR(I)
X=HALFA*(XSI+1)
H(1,7)=6.*X
H(2,9)=2.*X
H(2,12)=H(2,9)*X
H(2,14)=H(2,12)*X
H(3,8)=4.*X
H(3,11)=H(1,7)*X
DO 48 J=1,3
YJ=WBAR(J)
WJ=WBAR(J)
Y=HALFB*(YJ+1)
WINJ=XI*YJ
EVALUATE P(X,Y) AT XI,YJ
FIRST EVALUATE H AT XI,YJ
H(1,8)=2.*Y
H(1,11)=H(1,7)*Y
H(1,12)=H(1,8)*Y
H(1,14)=H(1,11)*Y
H(1,15)=H(1,12)*Y
H(1,16)=H(1,14)*Y
H(2,10)=6.*Y

```

C

C C


```

H(2,13)=H(1,11)
H(2,15)=H(2,13)*X
H(2,16)=H(2,15)*X
H(3, 9)=4.*Y
H(3,12)=H(2,9)*H(3,9)
H(3,13)=H(2,10)*Y
H(3,14)=H(1,8)*H(3,11)
H(3,15)=H(2,9)*H(3,13)
H(3,16)=H(1,14)*3.*X
C COMPUTE PRODUCT OF (H TRANSPOSE)X(O)X(H) X W1 X WJ
DO 40 I1=1,16
DO 40 JJ=1,16
HTDH(I1,JJ)=0.
DO 30 LL=1,3
DO 30 KK=1,3
HTDH(I1,JJ)=HTDH(I1,JJ)+H(LL,I1)*D(LL,KK)*H(KK,JJ)
30 CONTINUE
HTDH(I1,JJ)=W1*WJ*HTDH(I1,JJ)
40 CONTINUE
C ADD HTDH INTO TOTAL P MATRIX
DO 45 I1=1,16
DO 45 JJ=1,16
P(I1,JJ)=P(I1,JJ)+HTDH(I1,JJ)
45 CONTINUE
C COMPUTE (G TRANSPOSE)X(INTEGRAL P)X(G)
DO 60 I1=1,16
DO 60 JJ=1,16
DO 50 LL=1,16
DO 50 KK=1,16
STIFFM(I1,JJ)=STIFFM(I1,JJ)+G(LL,I1)*P(I1,KK)*G(KK,JJ)
50 CONTINUE
C STIFFNESS MATRIX FOR AN ELEMENT NOT ON WELDED EDGE
52 STIFFM(I1,JJ)=AB3ET*STIFFM(I1,JJ)
60 CONTINUE
C STIFFNESS MATRIX IS COMPLETE AND ALL 0'S ARE ZERO
RETURN
END

```



```

C SUBROUTINE STIFX(G,STIFFX,QX)
C ADDITIONAL TERMS FOR STIFFNESS MATRIX FOR AN ELEMENT WITH WELDED
C EDGE PARALLEL TO X AXIS
C DIMENSION G(16,16),QX(16),STIFFX(16,16)
C COMMON /IOA KPEAD,KWRITE,KPUNCH
C COMMON /GAUSS/ WEAR(3),XBAR(3),YBAR(3)
C COMMON /CONST/ M,N,A,B,CX,CY,RETA,THETAZ,ARBET,HALFA,HALFB,ASQ,ACU
13,RSQ,BCU3,CXB2,CYA2,BTHCX,ATHCY
C COMMON /EXTRA/ P(16,16),WF(16),FX(16),EKX(16,16)
C CALL FRASE (P,256,WF,16,QX,16,EKX,256)
V=R
FX(1)=0.
FX(2)=0.
FX(3)=1.
FX(5)=0.
FX(6)=2.*Y
FX(7)=0.
FX(10)=3.*Y*X
DO 150 JJ=1,3
WJ=WEAR(JJ)
CONST=ATHCY*WJ
X=HALFA*(XBAR(JJ)+1)
FX(4)=X
FX(8)=X*X
FX(9)=FX(5)*X
FX(11)=X*X*X
FX(12)=FX(9)*X
FX(13)=FX(11)*X
FX(14)=FX(12)*X
FX(15)=FX(13)*X
FX(16)=FX(15)*X
DO 120 KK=1,16
WF(KK)=WF(KK)+CONST*FX(KK)
DO 120 LL=1,16
P(LL,KK)=P(LL,KK)+WJ*FX(LL)*FX(KK)
120 CONTINUE

```



```

130 CONTINUE
D1 150 II=1,16
D1 150 JJ=1,16
DX(II)=OX(II)+G(JJ,II)*WF(JJ)
D0 140 KK=1,16
D0 140 LL=1,16
EKX(II,JJ)=EKX(II,JJ)+G(LL,II)*P(LL,KK)*G(KK,JJ)
140 CONTINUE
STIFEX(II,JJ)=CYA2*EKX(II,JJ)
150 CONTINUE
RETURN
END

```



```

C SUBROUTINE STIFF(G,STIFFY,QY)
C ADDITIONAL TERMS FOR STIFFNESS MATRIX FOR AN ELEMENT WITH WELDED
C EDGE PARALLEL TO Y AXIS
C DIMENSION G(16,16),QY(16),STIFFY(16,16)
C COMMON /IO/ KREAD,KWRITE,KPUNCH
C COMMON /GAUSS/ WBAR(3),XBAR(3),YBAR(3)
C COMMON /CONST/ M,N,A,B,CX,CY,BETA,THETAZ,ARBET,HALFA,HALFB,ASQ,ACU
18,BSQ,BCUB,CXB2,CYA2,RTHCX,ATHCY
C COMMON /EXTRA/ P(16,16),WF(16),FY(16),EKY(16,16)
C CALL EPASE (P,256,WF,16,QY,16,EKY,256)
X=A
FY(1)=0.
FY(2)=1.
FY(3)=0.
FY(5)=2.*X
FY(6)=0.
FY(7)=3.*X*X
FY(10)=0.
DO 80 JJ=1,3
  YJ=WBAR(JJ)
  CONST=RTHCX*YJ
  Y=HALFB*(YBAR(JJ)+1)
  FY(4)=Y
  FY(8)=FY(5)*Y
  FY(9)=Y*Y
  FY(11)=FY(7)*Y
  FY(12)=FY(8)*Y
  FY(13)=FY(9)*Y
  FY(14)=FY(11)*Y
  FY(15)=FY(12)*Y
  FY(16)=FY(14)*Y
DO 70 KK=1,15
  WF(KK)=WF(KK)+CONST*FY(KK)
DO 70 LL=1,16
  P(LL,KK)=P(LL,KK)+FY(LL)*FY(KK)*YJ
70 CONTINUE

```



```

80  CONTINUE
   DO 100 II=1,16
   DO 100 JJ=1,16
      QY(II)=QY(II)+G(JJ,II)*WF(JJ)
   DO 90 KK=1,15
   DO 90 LL=1,15
      EKX(II,JJ)=EKX(II,JJ)+G(LL,II)*P(LL,KK)*G(KK,JJ)
90  CONTINUE
      STIFFY(II,JJ)=CX32*EKX(II,JJ)
100 CONTINUE
   RETURN
   END

```



```

1 INDEX = IMASTR+LNUM-1
  DO 2 I = 1,NDE
    J = INTGR(INDEX)+I-1
    2 MNUM(I) = INTGR(J)
  C LOOP OVER ROWS OF ELQ AND OVER LOWER TRIANGLE OF ELEMENT K MATRIX
    DO 4 LROW = 1,NDE
      INDEX = IQM1+MNUM(LROW)
      C ASSEMBLE ELEMENT EQUIVALENT NODAL FORCE INTO Q VECTOR
        REAL(INDEX) = REAL(INDEX)+ELO(LROW)
      DO 4 LCOL = 1,LROW
        MROW = MNUM(LROW)
        MCOL = MNUM(LCOL)
        IF (MROW .GE. MCOL) GO TO 3
        MROW = MNUM(LCOL)
        MCOL = MNUM(LROW)
      C CALCULATE ABSOLUTE ADDRESS OF K(MROW,MCOL)
        3 INDEX = IKQUM1+MROW
        KADR = INTGR(INDEX)+MCOL
      C ASSEMBLE STIFFNESS COEFFICIENT
        4 REAL(KADR) = REAL(KADR)+ELK(LROW,LCOL)
      RETURN
    END

```

ASEM00037
 ASEM00038
 ASEM00039
 ASEM00040
 ASEM00041
 ASEM00042
 ASEM00043
 ASEM00044
 ASEM00045
 ASEM00046
 ASEM00047
 ASEM00048
 ASEM00049
 ASEM00050
 ASEM00051
 ASEM00052
 ASEM00053
 ASEM00054
 ASEM00055
 ASEM00056
 ASEM00057
 ASEM00058


```

SUBROUTINE BCON(REAL,INTGR)
C *****
C FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2
C AEROELASTIC AND STRUCTURES RESEARCH LABGRATORY
C MASSACHUSETTS INSTITUTE OF TECHNOLOGY
C *****
  DIMENSION REAL(2),INTGR(2)
  COMMON /IO/ KR, KW, KP, KT1, KT2, KT3
  COMMON /SIZE/ NET, NDT
  COMMON /BEGIN/ ICON,IKOUNT,ILNZ,IMASTR,IQ,IK
  COMMON /END/ LCON,LKOUNT,LLNZ,LMASTR,LQ,LK

  COMMON /DYNAM/ LAST,IFLAG,I,J,MROW,M,NEXT,NROW,ICOL,KADR,N,
2 INIT,MCOL,ILNZM1,IKOUM1,IQMI

C
C PRINT CONTROL
901 FORMAT(79H0DISPLACEMENT CONSTRAINTS HAVE BEEN APPLIED TO THE FOLLO
  IING DEGREES OF FREEDOM,/5X,7HDOF NO.,2X,12HDISPLACEMENT)
902 FORMAT(2X,110,2X,E10.3)
903 FORMAT(1X,72H*****ABOVE DOF NUMBER APPEARS TWICEBCON0020
  1 IN CONSTRAINT LIST,/1X,85HEXECUTION TERMINATED IN SUBROUTINE BCON0021
  2N DUE TO POSSIBILITY OF BOUNDARY CONDITION ERROR)
904 FORMAT(62H0YOUR STRUCTURE IS FLYING FREE. PLEASE CONSTRAIN IT NEXT3CON0023
  1 TIME.,/1X,28HEXECUTION TERMINATED IN BCON)
  IKOUM1 = IKOUNT-1
  ILNZM1 = ILNZ-1
  IQMI = IQ-1
C PRINT ENTRY MESSAGE
  WRITE (KW,901)
C ORDER THE CONSTRAINT ROW NUMBERS IN ASCENDING SEQUENCE
  LAST = LCON-1
  IF (LCON .GT. 0) GO TO 1
  WRITE (KW,904)
  STOP
  1 IFLAG = 0
  DO 2 I = ICON, LAST

```

BCON0001

BCON0002

BCON0003

BCON0004

BCON0005

BCON0006

BCON0007

BCON0008

BCON0009

BCON0010

BCON0011

BCON0012

BCON0013

BCON0014

BCON0015

BCON0016

BCON0017

BCON0018

BCON0019

BCON0020

BCON0021

BCON0022

BCON0023

BCON0024

BCON0025

BCON0026

BCON0027

BCON0028

BCON0029

BCON0030

BCON0031

BCON0032

BCON0033

BCON0034

BCON0035

BCON0036


```

IF (INTGR(I+1) .GE. INTGR(I)) GO TO 2
J = INTGR(I)
INTGR(I) = INTGR(I+1)
INTGR(I+1) = J
IFLAG = 1
2 CONTINUE
IF (IFLAG .EQ. 1) GO TO 1
C CHECK TO SEE IF ANY ROW NUMBERS HAVE BEEN ENTERED IN CONSTRAINT
C VECTOR - ABORT THE RUN IF NONE HAVE BEEN
J = 0
DO 100 I = ICON,LCON
100 J = J+INTGR(I)
IF (J .GT. 0) GO TO 200
WRITE (KW,904)
STOP
C OUTPUT CONSTRAINT LIST
200 DO 4 I = ICON,LCON
IF (INTGR(I) .EQ. 0) GO TO 4
C CHECK FOR REPEATED DOF AFTER 1ST ONE
IF (I .EQ. ICON) GO TO 3
IF (INTGR(I) .NE. INTGR(I-1)) GO TO 3
WRITE (KW,903)
STOP
3 J = IQM1+INTGR(I)
WRITE (KW,902) INTGR(I), REAL(J)
4 CONTINUE
C LOOP OVER CONSTRAINED DOF FOR MODIFICATION OF COMPLETELY ASSEMBLED
C FORCE VECTOR
DO 71 I = ICON,LCON
IF (INTGR(I) .EQ. 0) GO TO 71
C CHECK IF PRESCRIBED DISPLACEMENT = 0 -- IF IT DOES, SKIP FORCE VECTOR
MROW = INTGR(I)
M = IQM1+MROW
IF (REAL(M) .EQ. 0.) GO TO 71
C DISPL .NE. 0 -- LOOP OVER ALL ROWS TO MODIFY FORCE VECTOR -- SKIP
C CONSTRAINED ROWS (CONTROLLED BY VALUE OF NEXT)

```

BCON0037
 BCON0038
 BCON0039
 BCON0040
 BCON0041
 BCON0042
 BCON0043
 BCON0044
 BCON0045
 BCON0046
 BCON0047
 BCON0048
 BCON0049
 BCON0050
 BCON0051
 BCON0052
 BCON0053
 BCON0054
 BCON0055
 BCON0056
 BCON0057
 BCON0058
 BCON0059
 BCON0060
 BCON0061
 BCON0062
 BCON0063
 BCON0064
 BCON0065
 BCON0066
 BCON0067
 BCON0068
 BCON0069
 BCON0070
 BCON0071
 BCON0072


```

NEXT = ICON
DO 7 NROW = 1, NDT
  IF (NROW .NE. INTEGR(NEXT)) GO TO 5
  NEXT = NEXT+1
  GO TO 7
5 IF (MROW .GT. NROW) GO TO 51
C CHECK FOR COUPLING OF ROW NROW WITH COL MROW
  J = ILNZM1+NROW
  IF (INTEGR(J) .GT. MROW) GO TO 7
  IROW = NROW
  ICOL = MROW
  GO TO 6
C CHECK FOR COUPLING OF ROW MROW WITH COL NROW
51 J = ILNZM1+MROW
  IF (INTEGR(J) .GT. NROW) GO TO 7
  IROW = MROW
  ICOL = NROW
C SUBTRACT K*(PRESCR DISPL) FROM FORCE VECTOR
6 KADR = IKOUM1+IROW
  KADR = INTEGR(KADR)+ICOL
  N = IQM1+NROW
  REAL(N) = REAL(N)-REAL(KADR)*REAL(M)
7 CONTINUE
71 CONTINUE
C LOOP OVER CONSTRAINED ROWS TO DECOUPLE THEM FROM REST OF K MATRIX
DO 11 I = ICON, LCON
  IF (INTEGR(I) .EQ. 0) GO TO 11
  MROW = INTEGR(I)
  INIT = ILNZM1+MROW
  INIT = INTEGR(INIT)
C SET ROW = 0
  M = IKOUM1+MROW
  M = INTEGR(M)
DO 8 MCOL = INIT, MROW
  KADR = M+MCOL
8 REAL(KADR) = 0.

```

BCON0073
 BCON0074
 BCON0075
 BCON0076
 BCON0077
 BCON0078
 BCON0079
 BCON0080
 BCON0081
 BCON0082
 BCON0083
 BCON0084
 BCON0085
 BCON0086
 BCON0087
 BCON0088
 BCON0089
 BCON0090
 BCON0091
 BCON0092
 BCON0093
 BCON0094
 BCON0095
 BCON0096
 BCON0097
 BCON0098
 BCON0099
 BCON0100
 BCON0101
 BCON0102
 BCON0103
 BCON0104
 BCON0105
 BCON0106
 BCON0107
 BCON0108


```

C SET COLUMN = 0 IN ROWS WHOSE LNZE COL NO IS .LE. MROW -- SKIP THIS
C SECTION IF MROW IS THE LAST ROW
IF (MROW .EQ. NDT) GO TO 10
INIT = MROW+1
DO 9 MROW = INIT, NDT
  N = ILN2M1+MROW
  IF (INTGR(N) .GT. MROW) GO TO 9
  KADR = IKOUM1+MROW
  KADR = INTGR(KADR)+MROW
  REAL(KADR) = 0.
9 CONTINUE
C SET DIAGONAL ENTRY = 1
10 KADR = M+MROW
  REAL(KADR) = 1.
11 CONTINUE
  RETURN
END
BCON0109
BCON0110
BCON0111
BCON0112
BCON0113
BCON0114
BCON0115
BCON0116
BCON0117
BCON0118
BCON0119
BCON0120
BCON0121
BCON0122
BCON0123
BCON0124
BCON0125

```



```

C PRINT ENTRY MESSAGE
  1 WRITE (KW,901)
    IKOUM1 = IKOUNT-1
    ILNZM1 = ILNZ-1
C INITIALIZE AT 1ST NONZERO ENTRY IN CONSTRAINT VECTOR
  DO 2 I = ICON,LCUN
    IF (INTGR(I) .EQ. 0) GO TO 2
    GO TO 3
  2 CONTINUE
  3 NEXT = 1
    IF (INTGR(NEXT) .EQ. 1) NEXT = NEXT+1
C INITIALIZE ERROR PARAMETERS
  JROW = 1
  MPD = 0
  TEST = 1.
C DO FIRST ROW AS SPECIAL CASE
  IF (REAL(IK) .EQ. 0.) GO TO 14
  IF (REAL(IK) .GT. 0.) GO TO 4
C NPD MESSAGE AND FLAG
  WRITE (KW,902) JROW
  MPD = 1
C LOOP OVER REMAINING ROWS
  4 DO 13 MROW = 2,NDT
C CHECK FOR CONSTRAINED ROW - SKIP IF FOUND AND RESET FOR THE NEXT ONE
  IF (MROW .NE. INTGR(NEXT)) GO TO 5
  NEXT = NEXT+1
  GO TO 13
C FREE ROW - FACTOR FROM LN2 COL NO TO ROW NO
  5 M = ILNZM1+MROW
  M = INTGR(M)
  MM = IKOUM1+MROW
  MM = INTGR(MM)
  DO 12 MCOL = M,MROW
    SUM = 0.
  NN = IKOUM1+MCOL
  NN = INTGR(NN)

```

FACT0037
 FACT0038
 FACT0039
 FACT0040
 FACT0041
 FACT0042
 FACT0043
 FACT0044
 FACT0045
 FACT0046
 FACT0047
 FACT0048
 FACT0049
 FACT0050
 FACT0051
 FACT0052
 FACT0053
 FACT0054
 FACT0055
 FACT0056
 FACT0057
 FACT0058
 FACT0059
 FACT0060
 FACT0061
 FACT0062
 FACT0063
 FACT0064
 FACT0065
 FACT0066
 FACT0067
 FACT0068
 FACT0069
 FACT0070
 FACT0071
 FACT0072


```

C LNZE IS A SPECIAL CASE - NO SUM REQUIRED
  IF (MCOL.EQ. M) GO TO 7
C START SUM FROM GREATEST OF MROW OR ROW 'MCOL' LN2 COL NOS
  INIT = M
  N = ILN2M1+MCOL
  IF (INTGR(N).GT. M) INIT = INTGR(N)
C NO SUM IF ROW 'MCOL' HAS LEADING ZEROS UP TO THE DIAGONAL
  IF (INIT.EQ. MCOL) GO TO 7
C ACCUMULATE THE SUM
  LAST = MCOL-1
  DO 6 J = INIT, LAST
    KADRM = MM+J
    KADRN = NN+J
    JJ = IKOUM1+J
    JJ = INTGR(JJ)+J
    6 SUM = SUM+REAL(JJ)*REAL(KADRM)*REAL(KADRN)
C BRANCH TO SPECIAL ALGORITHM FOR DIAGONAL ENTRIES
  7 IF (MCOL.EQ. MROW) GO TO 8
C FOR OFF-DIAGONAL ENTRIES:
  KADR = MM+MCOL
  NN = NN+MCOL
  REAL(KADR) = (REAL(KADR)-SUM)/REAL(NN)
  GO TO 12
C DIAGONAL ENTRY - TEST FOR SINGULARITY AND SEMI-DEFINITENESS
  8 MM = MM+MROW
  IF (REAL(MM)-SUM.NE. 0.) GO TO 9
  JROW = MROW
  GO TO 14
  9 IF (REAL(MM)-SUM.GT. 0.) GO TO 10
  WRITE (KW,902) MROW
  MPD = 1
C CALCULATE ROUNDING ERROR
  10 TESTR = ABS((REAL(MM)-SUM)/REAL(MM))
  IF (TESTR.GE. TEST) GO TO 11
  TEST = TESTR
  IROW = MROW

```

FACT0073
 FACT0074
 FACT0075
 FACT0076
 FACT0077
 FACT0078
 FACT0079
 FACT0080
 FACT0081
 FACT0082
 FACT0083
 FACT0084
 FACT0085
 FACT0086
 FACT0087
 FACT0088
 FACT0089
 FACT0090
 FACT0091
 FACT0092
 FACT0093
 FACT0094
 FACT0095
 FACT0096
 FACT0097
 FACT0098
 FACT0099
 FACT0100
 FACT0101
 FACT0102
 FACT0103
 FACT0104
 FACT0105
 FACT0106
 FACT0107
 FACT0108

FACT0109
 FACT0110
 FACT0111
 FACT0112
 FACT0113
 FACT0114
 FACT0115
 FACT0116
 FACT0117
 FACT0118
 FACT0119
 FACT0120
 FACT0121
 FACT0122
 FACT0123
 FACT0124
 FACT0125
 FACT0126

```

C EVALUATE DIAGONAL ENTRY
  11 REAL(MM) = REAL(MM)-SUM
  12 CONTINUE
  13 CONTINUE
C SEMI-DEFINITENESS CHECKS AND ROUNDING ERROR OUTPUT
  IF (MPD .EQ. 0) WRITE (KW,903)
  IF (MPD .EQ. 1 .AND. NPD .EQ. 1) WRITE (KW,905)
  IF (MPD .EQ. 1 .AND. NPD .EQ. 0) WRITE (KW,904)
  IERR = -1.00001*ALOG10(TEST)
  WRITE (KW,907) IROW,IERR
  IF (IERR .GT. 5) WRITE (KW,908)
  IF (MPD .EQ. 1 .AND. NPD .EQ. 1) STOP
  IF (IERR .GT. 5) STOP
  RETURN
C SINGULAR MATRIX
  14 WRITE (KW,906) JROW
    STOP
  END

```



```

C ***** SUBROUTINE ORK(LENGTH,REAL,INTGR) ORK 0001
C ***** C FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2 ORK 0002
C ***** C AEROELASTIC AND STRUCTURES RESEARCH LABORATORY ORK 0003
C ***** C MASSACHUSETTS INSTITUTE OF TECHNOLOGY ORK 0004
C ***** C ***** ORK 0005
C ***** C ***** ORK 0006
C ***** C THIS SUBROUTINE CREATES ORK 0007
C ***** C 1) THE LN2 VECTOR WHICH HOLDS THE COLUMN NUMBER ORK 0008
C ***** C OF THE LEADING NON-ZERO ENTRY IN EACH ROW OF ORK 0009
C ***** C THE ASSEMBLED "K" MATRIX ORK 0010
C ***** C 2) THE ADDRESS COUNT VECTOR WHICH HOLDS THE ABSOLUTE ORK 0011
C ***** C ADDRESS OF THE DIAGONAL ENTRY FOR EACH ROW OF THE ORK 0012
C ***** C ASSEMBLED K MATRIX, MINUS THE ROW NUMBER, AND ORK 0013
C ***** C CHECKS THAT K FITS IN THE /DATA/ VECTOR ORK 0014
C ***** C DIMENSION REAL(2),INTGR(2) ORK 0015
C ***** C COMMON /IO/ KP, KW, KP, KT1, KT2, KT3 ORK 0016
C ***** C COMMON /SIZE/ NET, NDT ORK 0017
C ***** C COMMON /BEGIN/ ICON,IKOUNT,ILNZ,IMASTR,IQ,IK ORK 0018
C ***** C COMMON /END/ LCON,LKOUNT,LLNZ,LMASTR,LQ,LK ORK 0019
C ***** C ORK 0020
C ***** C COMMON /DYNAM/ ILNZM1,IMSTM1,IKOUM1,NETM1,IMSTP1,IROW,MSUB,LNUM, ORK 0021
C ***** C 2 MADPR,NDE,ISMAIL,JDOF,INDEX,NENTRY,DENS,I,J ORK 0022
C ***** C ORK 0023
C ***** C ORK 0024
C ***** C PRINT CONTROL ORK 0025
C ***** C 100 FORMAT(49H0THE LENGTH OF THE "DATA" VECTOR FOR THIS CASE IS,I10,I4ORK 0026
C ***** C 1H WHICH EXCEEDS,I10,49H=THE MAXIMUM ALLOWED IN THE DIMENSION STATORK 0027
C ***** C 2MENT./39H EXECUTION TERMINATED IN SUBROUTINE ORK) ORK 0028
C ***** C 200 FORMAT(5X,3HROW,2X,13HLNZE COL. NO.,2X,18HABS. ADR. OF DIAG.) ORK 0029
C ***** C 300 FORMAT(I9,I10,I12) ORK 0030
C ***** C 400 FORMAT(10H0THERE ARE,I10,68H NON-ZERO ENTRIES IN "K". IF "K" WERE ORK 0031
C ***** C 1FULLY POPULATED THERE WOULD BE,I10, 9H ENTRIES.,/,20X,15HTHE DENSIO ORK 0032
C ***** C 2TY IS ,E15.6) ORK 0033
C ***** C ILNZM1=ILNZ-1 ORK 0034
C ***** C IMSTM1=IMASTR-1 ORK 0035
C ***** C IKOUM1=IKOUST-1 ORK 0036
C ***** C NETM1=NET-1

```



```

IMSTPI=IMASTR+1
C SET EACH LNZ COLUMN NO = ROW NO (DIAGONAL MATRIX)
DO 30 IROW=1,NDT
  MSUB=ILNZM1+IROW
  30 INTGR(MSUB)=IROW
C EXAMINE MASTER ASSEMBLY LIST, ONE ELEMENT AT A TIME, TO CREATE
C THE LNZ VECTOR
DO 20 LNUM=1,NET
  MADDR = IMSTM1+LNUM
  MADDR = INTGR(MADDR)-1
  C CALCULATE NO. DOF IN THE ELEMENT BY DIFFERENCING POINTERS, OR ...
  I = IMASTR+LNUM
  IF(LNUM.EQ.NET) GO TO 3
  NDE = INTGR(I)-INTGR(I-1)
  GO TO 4
  C ... BEGIN BY ASSUMING THE LIST IS FILLED, FOR LAST ELEMENT
  3 NDE = LMASTR-INTGR(I-1)+1
  C INITIALIZE SMALLEST DOF NO. AT LARGEST POSSIBLE VALUE
  4 ISMALL=NDT
  C FIND SMALLEST MASTR NUMBER FOR THIS ELEMENT
  DO 5 JDOF=1,NDE
    INDEX=MADDR+JDOF
    IF(INTGR(INDEX).GT.ISMALL) GO TO 5
  C DISCONTINUE SEARCH IF A ZERO IS FOUND, INDICATING EXCESS STORAGE
  C AND PREMATURE END OF LIST FOR LAST ELEMENT
  IF(INTGR(INDEX).EQ.0) GO TO 6
  ISMALL=INTGR(INDEX)
  5 CONTINUE
  C FIND COLUMN NUMBER OF LEADING NON-ZERO ENTRY IN ROW
  6 DO 10 JDOF=1,NDE
    INDEX=MADDR+JDOF
    INDEX=ILNZM1+INTGR(INDEX)
  C CHANGE LNZ COL NO ONLY IF NEW ONE IS LESS THAN OLD ONE
  IF(INTGR(INDEX).LT.ISMALL) GO TO 8
  INTGR(INDEX)=ISMALL
  GO TO 10

```

```

ORK 0037
ORK 0038
ORK 0039
ORK 0040
ORK 0041
ORK 0042
ORK 0043
ORK 0044
ORK 0045
ORK 0046
ORK 0047
ORK 0048
ORK 0049
ORK 0050
ORK 0051
ORK 0052
ORK 0053
ORK 0054
ORK 0055
ORK 0056
ORK 0057
ORK 0058
ORK 0059
ORK 0060
ORK 0061
ORK 0062
ORK 0063
ORK 0064
ORK 0065
ORK 0066
ORK 0067
ORK 0068
ORK 0069
ORK 0070
ORK 0071
ORK 0072

```



```

C DISCONTINUE OPERATION IF EXCESS STORAGE IS DISCOVERED
8  IF (INTGR(INDEX).EQ.0) GO TO 20
10 CONTINUE
20 CONTINUE
C CREATE ADDRESS COUNT VECTOR
  INTGR(IKOUNT) = IK
  INDEX=IKOUNT
  DO 40 IROW=2,NDT
    I = ILNZM1+IROW
    INTGR(INDEX+1) = INTGR(INDEX)+IROW+1-INTGR(I)
    INDEX=INDEX+1
40
C ADDRESS COUNT VECTOR NOW CONTAINS ABSOLUTE ADDRESS ONLY FOR THE
C DIAGONAL ENTRIES, AND THUS INTGR(LKOUNT) = LK EXACTLY
  IF (INTGR(LKOUNT) .LE. LENGTH) GO TO 50
  WRITE (KW,100) INTGR(LKOUNT), LENGTH
  STOP
50  LK = INTGR(LKOUNT)
  WRITE(KW,200)
  DO 60 IROW=1,NDT
    I = ILNZM1+IROW
    J = IKOUM1+IROW
    WRITE (KW,300) IROW, INTGR(I), INTGR(J)
    C REPLACE THE ABS. ADDRESS OF DIAG. BY (ABS. ADDRESS - ROW NO.)
    INTGR(J) = INTGR(J)-IROW
    CONTINUE
60  NENTRY = INTGR(LKOUNT)+NDT-IK+1
    INDEX = (NDT*(NDT+1))/2
    DENS = FLOAT(NENTRY)/FLOAT(INDEX)
    WRITE (KW,400) NENTRY, INDEX, DENS
    C ZERO THE FORCE/DISPLACEMENT VECTOR AND THE K MATRIX BLOCK
    DO 70 I=IQ,LK
      REAL(I)=0.
70  RETURN
    END

```

ORK 0073
 ORK 0074
 ORK 0075
 ORK 0076
 ORK 0077
 ORK 0078
 ORK 0079
 ORK 0080
 ORK 0081
 ORK 0082
 ORK 0083
 ORK 0084
 ORK 0085
 ORK 0086
 ORK 0087
 ORK 0088
 ORK 0089
 ORK 0090
 ORK 0091
 ORK 0092
 ORK 0093
 ORK 0094
 ORK 0095
 ORK 0096
 ORK 0097
 ORK 0098
 ORK 0099
 ORK 0100
 ORK 0101
 ORK 0102
 ORK 0103
 ORK 0104
 ORK 0105
 ORK 0106


```

SUBROUTINE SETUP(LENGTH,NCON,MASTR,REAL,INTGR)
*****
C FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2
C AEROELASTIC AND STRUCTURES RESEARCH LABORATORY
C MASSACHUSETTS INSTITUTE OF TECHNOLOGY
*****
DIMENSION REAL(2),INTGR(2)
DIMENSION II(6), LI(6), KD(6)
COMMON /IO/ KR, KW, KP, KT1, KT2, KT3
COMMON /SIZE/ NET, NDT
COMMON /BEGIN/ ICON,IKOUNT,ILNZ,IMASTR,IQ,IK
COMMON /END/ LCON,LKOUNT,LLNZ,LMASTR,LQ,LK

COMMON /DYNAM/ I, TR, DENS
EQUIVALENCE (ICON, II(1)), (LCON, LI(1)), (KR, KD(1))

C
C PRINT CONTROL
901 FORMAT (IHO,53X,11HSETUP ENTRY,/,42X,35HUSER SPECIFICATIONS TO FEASETU0018
1BL SYSTEM,/,1HO,51X,16HI/O DEVICE CODES,/,27X,6HREADER,5X,7HPRINTSETU0019
2R,2X,10HCARD PUNCH,7X,5HTAPE1,7X,5HTAPE2,7X,5HTAPE3,/,21X,6(2X,110SETU0020
3),/,1HO,53X,12HPROBLEM SIZE,/,42X,25HTOTAL NUMBER OF ELEMENTS=,110SETU0021
4,/,42X,25HTOTAL DEGREES OF FREEDOM=,110,/,1HO,51X,16HENTRY PARAMETSETU0022
5ERS,/,39X,32HASSUMED LENGTH OF /DATA/ VECTOR=,110,/,32X,45HNUMBER SETU0023
6OF DISPLACEMENT CONSTRAINTS REQUESTED=,110,/,33X,44HNUMBER OF WORDSETU0024
7S REQUESTED FOR ASSEMBLY LIST=,110)
902 FORMAT (1HO,43X,31H/DATA/ VECTOR ADDRESS INDEX MAP,/,22X,11HCONSTRSETU0026
1AINTS,2X,10HDC ABS ADR,1X,11HLNZE COL NO,1X,11HASMBLY LIST,2X,10HQSETU0027
2/U VECTOR,4X,8HK MATRIX,/,16X,5HBEGIN,6(2X,110),/,18X,3HEND,5(2X,1SETU0028
310),11X,1H+,/,1HO,36H+ LK IS CALCULATED BY SUBROUTINE CRK)
903 FORMAT (40H0STORAGE EXCEEDS LENGTH OF /DATA/ VECTOR,/,1X,34HLENGTHSETU0030
1 SUGGESTED FOR THIS PROBLEM=,112,6H WORDS,/,1X,40HEXECUTION TERMINSETU0031
2ATED IN SUBROUTINE SETUP)
*****
C REMOVE THIS FORMAT AND WRITE STATEMENT INDICATED BELOW IF FEABL
C HEADING IS NOT DESIRED
1001 FORMAT(1H1/3(1H ,92(1HX))/,4(5H XXXX,84X,4HXXXXX//),5H XXXX,7X,10(1HSETU0036

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1F),5X,10(1HE),9X,2HAA,9X,8(1HB),7X,3HLLL,14X,4HXXX/5H XXXX,7X,10(SETU0037
21HF),5X,10(1HE),8X,4HAAA,8X,9(1HB),6X,3HLLL,14X,4HXXX/5H XXXX,7X,SETU0038
3,3HFFF,12X,3HEEE,14X,6(1HA),7X,3HB8B,3X,4HB8B,5X,3HLLL,14X,4HXXX/5H XXXX,7X,SETU0039
4/5H XXXX,7X,3HFFF,12X,3HEEE,13X,3HAAA,2X,3HAAA,6X,3HB8B,4X,3HB8B,5X,SETU0040
5X,3HLLL,14X,4HXXX/5H XXXX,7X,3HFFF,12X,3HEEE,12X,3HAAA,4X,3HAAA,5X,SETU0041
6X,3HB8B,4X,3HB8B,5X,3HLLL,14X,4HXXX/5H XXXX,7X,3HFFF,12X,3HEEE,12X,SETU0042
7X,3HAAA,4X,3HAAA,5X,3HB8B,3X,4HB8B,5X,3HLLL,14X,4HXXX/5H XXXX,7X,SETU0043
8,8(1HF),7X,8(1HE),7X,3HAAA,4X,3HAAA,5X,9(1HB),6X,3HLLL,14X,4HXXX/5H XXXX,7X,SETU0044
95H XXXX,7X,8(1HF),7X,8(1HE),7X,10(1HA),5X,9(1HB),6X,3HLLL,14X,4HXXX/5H XXXX,7X,SETU0045
AXX/5H XXXX,7X,3HFFF,12X,3HEEE,12X,10(1HA),5X,3HB8B,3X,4HB8B,5X,3HSETU0046
BLLL,14X,4HXXX/5H XXXX,7X,3HFFF,12X,3HEEE,12X,3HAAA,4X,3HAAA,5X,3HSETU0047
C8B8,4X,3HB8B,5X,3HLLL,14X,4HXXX/5H XXXX,7X,3HFFF,12X,3HEEE,12X,3HSETU0048
DAAA,4X,3HAAA,5X,3HB8B,3X,4HB8B,5X,3HLLL,14X,4HXXX/5H XXXX,7X,3HSETU0049
EFF,12X,10(1HE),5X,3HAAA,4X,3HAAA,5X,9(1HB),6X,10(1HL),7X,4HXXX/5HSETU0050
F XXXX,7X,3HFFF,12X,10(1HE),5X,3HAAA,4X,3HAAA,5X,8(1HB),7X,10(1HL),SETU0051
G7X,4HXXX/4(5H XXXX,84X,4HXXX/),5H XXXX,24X,37FINITE ELEMENT ANASETU0052
HLYSIS BASIC LIBRARY,23X,4HXXX/4(5H XXXX,84X,4HXXX/),3(1H,92(1HXXSETU0053
I/),1HI)
SETU0054
*****
C PRINT ENTRY MESSAGE
SETU0055
*****
C REMOVE THIS WRITE STATEMENT IF FEABL HEADING IS NOT DESIRED
SETU0056
*****
C WRITE (KW,1001)
SETU0057
*****
C WRITE (KW,901) (KD(I), I = 1,6), NET, NDT, LENGTH, NCON, MASTRL
SETU0058
*****
C CALCULATE ADDRESS INDEX VALUES
SETU0059
*****
C ICON = 1
SETU0060
*****
C LCON = NCON
SETU0061
*****
C IKOUNT = LCON+1
SETU0062
*****
C LKOUNT = LCON+NDT
SETU0063
*****
C ILNZ = LKOUNT+1
SETU0064
*****
C LLNZ = LKOUNT+NDT
SETU0065
*****
C IMASTR = LLNZ+1
SETU0066
*****
C LMASTR = LLNZ+MASTRL
SETU0067
*****
C IQ = LMASTR+1
SETU0068
*****
C LQ = LMASTR+NDT
SETU0069
*****

```



```

      IK = LQ+1
C PRINT INDEX MAP
      WRITE (KW,902) (II(I), I = 1,6), (LI(I), I = 1,5)
C STORAGE BOUNDS TEST
      IF (LMASTR.LE. LENGTH) GO TO 1
C STORAGE EXCEEDED - ESTIMATE REQUIRED LENGTH BASED ON LOWER TRIANGLE
C ESTIMATED POPULATION FACTOR
      TR = (NDT*(NDT+1))/2
      DENS = 0.5
      IF (NDT.GT. 200) DENS=0.3
      IF (NDT.GT. 500) DENS=0.2
      IF (NDT.GT. 1500) DENS = 0.15
      IF (NDT.GT. 2000) DENS = 0.10
      TR = TR*DENS
      LENGTH = LQ+TR
      WRITE (KW,903) LENGTH
      STOP
C ENOUGH STORAGE EXISTS TO GO THRU ORK
      1 DO 2 I = ICON,LCON
      2 INTGR(I) = 0
      RETURN
      END

```

```

SETU00073
SETU00074
SETU00075
SETU00076
SETU00077
SETU00078
SETU00079
SETU00080
SETU00081
SETU00082
SETU00083
SETU00084
SETU00085
SETU00086
SETU00087
SETU00088
SETU00089
SETU00090
SETU00091
SETU00092
SETU00093
SETU00094

```



```

C ***** SUBROUTINE SIMULQ(ENERGY,REAL,INTGR) SIMU0001
C ***** C FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2 SIMU0002
C ***** C AEROELASTIC AND STRUCTURES RESEARCH LABCRATORY SIMU0003
C ***** C MASSACHUSETTS INSTITUTE OF TECHNOLOGY SIMU0004
C ***** C ***** SIMU0005
C ***** C ***** SIMU0006
C ***** C ***** SIMU0007
C ***** C ***** SIMU0008
C ***** C ***** SIMU0009
C ***** C ***** SIMU0010
C ***** C ***** SIMU0011
C ***** C ***** SIMU0012
C ***** C ***** SIMU0013
C ***** C ***** SIMU0014
C ***** C ***** SIMU0015
C ***** C ***** SIMU0016
C ***** C ***** SIMU0017
C ***** C ***** SIMU0018
C ***** C ***** SIMU0019
C ***** C ***** SIMU0020
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C ***** C ***** SIMU0023
C ***** C ***** SIMU0024
C ***** C ***** SIMU0025
C ***** C ***** SIMU0026
C ***** C ***** SIMU0027
C ***** C ***** SIMU0028
C ***** C ***** SIMU0029
C ***** C ***** SIMU0030
C ***** C ***** SIMU0031
C ***** C ***** SIMU0032
C ***** C ***** SIMU0033
C ***** C ***** SIMU0034
C ***** C ***** SIMU0035
C ***** C ***** SIMU0036

SUBROUTINE SIMULQ(ENERGY,REAL,INTGR)
C *****
C FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2
C AEROELASTIC AND STRUCTURES RESEARCH LABCRATORY
C MASSACHUSETTS INSTITUTE OF TECHNOLOGY
C *****
  DIMENSION REAL(2),INTGR(2)
  COMMON /IO/ KR, KW, KP, KT1, KT2, KT3
  COMMON /SIZE/ NET, NDT
  COMMON /BEGIN/ ICUN, IKOUNT, ILNZ, IMASTR, IQ, IK
  COMMON /END/ LCON, LKOUNT, LLNZ, LMASTR, LQ, LK

  COMMON /DYNAM/ IKOUM1, ILNZM1, IQM1, IFLAG, MOST, LEFT, JBEG, NBEG, I,
2 JEND, NEND, J, N, NEXT, MROW, SUM, LAST, M, MM, KADR, INIT

C PRINT CONTROL
901 FORMAT (1H0,53X,18HSIMULQ ENTRY POINT,/,38H PRESCRIBED FORCE/DISPL
2ACEMENT VECTOR: )
902 FORMAT (30H0DISPLACEMENT SOLUTION VECTOR: )
903 FORMAT (1H0,2X,3HROW,10I12)
904 FORMAT (6H VALUE,10(2X,E10.3))
905 FORMAT (32H0STRAIN ENERGY IN THE STRUCTURE=,E10.3)
  IKOUM1 = IKOUNT-1
  ILNZM1 = ILNZ-1
  IQM1 = IQ-1
  C PRINT ENTRY MESSAGE, SET PRINT SECTION CONTROL FLAG
  WRITE (KW,901)
  IFLAG = 0
  C PRINT SECTION: DIVIDE Q VECTOR INTO GROUPS OF TEN
  C PLUS A POSSIBLE REMAINDER
  MOST = NDT/10
  LEFT = NDT-10*MOST
  JBEG = 1
  NBEG = IQ
  C RE-ENTRY POINT FOR SOLUTION OUTPUT
  1 IF (NDT.LT. 10) GO TO 3

```



```

DO 2 I = 1,MOST
JBEG = 1+10*(I-1)
JEND = JBEG+9
NREG = 10+JBEG-1
NEND = NBEG+9
WRITE (KW,903) (J, J = JBEG,JEND)
2 WRITE (KW,904) (REAL(N), N = NBEG,NEND)
C CHECK FOR EXISTENCE OF REMAINDER
IF (LEFT.EQ. 0) GO TO 4
JBEG = JEND+1
NBEG = NEND+1
3 WRITE (KW,903) (J, J = JBEG,NDT)
WRITE (KW,904) (REAL(N), N = NBEG,LQ)
C CHECK CONTROL FLAG
4 IF (IFLAG.EQ. 0) GO TO 5
WRITE (KW,905) ENERGY
RETURN
C FORWARD SOLUTION - NO DIVISIONS, SO SKIP FIRST ROW ENTIRELY
C INITIALIZE CONSTRAINT VECTOR AT 1ST NCNZERO ENTRY
5 DO 6 I = ICON,LCON
IF (INTGR(I).EQ. 0) GO TO 6
GO TO 7
6 CONTINUE
7 NEXT = I
IF (INTGR(NEXT).EQ. 1) NEXT = NEXT+1
C SOLVE (A)(R) = (Q)
DO 10 MROW = 2,NDT
C CHECK FOR CONSTRAINT TO SKIP ROW
IF (MROW.NE. INTGR(NEXT)) GO TO 8
C UPDATE NEXT
NEXT = NEXT+1
GO TO 10
C INITIALIZE SUM AND LOOP LIMITS FOR SUM
8 SUM = 0.
LAST = MROW-1
M = ILNZ+LAST

```

SIMU0037
SIMU0038
SIMU0039
SIMU0040
SIMU0041
SIMU0042
SIMU0043
SIMU0044
SIMU0045
SIMU0046
SIMU0047
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SIMU0058
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SIMU0065
SIMU0066
SIMU0067
SIMU0068
SIMU0069
SIMU0070
SIMU0071
SIMU0072


```

M = INTEGR(M)
MM = IKOUNT+LAST
MM = INTEGR(MM)
DO 9 J = M, LAST
  KADR = MM+J
  N = IQM1+J
    9 SUM = SUM+REAL(KADR)*REAL(N)
  C SUBTRACT SUM FROM Q
  N = IQM1+MROW
    REAL(N) = REAL(N)-SUM
  10 CONTINUE
  C SOLVE (D)(P) = (R) AND CALCULATE ENERGY
  N = IQ-1
  ENERGY = 0.
  DO 11 MROW = 1, NDT
    KADR = IKQUM1+MROW
    KADR = INTEGR(KADR)+MROW
    N = N+1
    REAL(N) = REAL(N)/REAL(KADR)
    11 ENERGY = ENERGY+REAL(KADR)*(REAL(N)**2)
    ENERGY = 0.5*ENERGY
  C BACK SOLUTION - NO DIVISIONS, SO SKIP LAST ROW ENTIRELY
  NEXT = LCON
  IF (INTEGR(NEXT) .EQ. NDT) NEXT = NEXT-1
  C LOOP BACKWARDS OVER REMAINING ROWS
  DO 14 I = 2, NDT
    MROW = NDT+1-I
  C CHECK FOR CONSTRAINT TO SKIP ROW
  IF (MROW .NE. INTEGR(NEXT)) GO TO 12
  C UPDATE NEXT
  NEXT = NEXT-1
  IF (NEXT .LT. ICON) NEXT = LCON
  GO TO 14
  C INITIALIZE SUM AND LOWER LOOP LIMIT
  12 SUM = 0.
  INIT = MROW+1

```

SIMU0073
 SIMU0074
 SIMU0075
 SIMU0076
 SIMU0077
 SIMU0078
 SIMU0079
 SIMU0080
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 SIMU0082
 SIMU0083
 SIMU0084
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 SIMU0088
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 SIMU0094
 SIMU0095
 SIMU0096
 SIMU0097
 SIMU0098
 SIMU0099
 SIMU0100
 SIMU0101
 SIMU0102
 SIMU0103
 SIMU0104
 SIMU0105
 SIMU0106
 SIMU0107
 SIMU0108


```

DO 13 J = INIT, NDT
C CHECK IF LN2 COL NO OF ROW J EXCEEDS MROW - IF SO, SKIP
N = ILNZM1+J
IF (INTGR(N) .GT. MROW) GO TO 13
KADR = IKOUM1+J
KADR = INTGR(KADR)+MROW
N = IQM1+J
SUM = SUM+REAL(KADR)*REAL(N)
13 CONTINUE
C SUBTRACT SUM FROM Q
N = IQM1+MROW
REAL(N) = REAL(N)-SUM
14 CONTINUE
C PRINT SOLUTION HEADING, RESET CONTROL FLAG AND
C BRANCH TO OUTPUT SECTION
WRITE (KW, 902)
IFLAG = 1
GO TO 1
END
SIMU0109
SIMU0110
SIMU0111
SIMU0112
SIMU0113
SIMU0114
SIMU0115
SIMU0116
SIMU0117
SIMU0118
SIMU0119
SIMU0120
SIMU0121
SIMU0122
SIMU0123
SIMU0124
SIMU0125
SIMU0126
SIMU0127

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```

SUBROUTINE XTRACT(LNUM,NDE,ELQ,REAL,INTGR)
C*****
C FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2
C AEROELASTIC AND STRUCTURES RESEARCH LABORATORY
C MASSACHUSETTS INSTITUTE OF TECHNOLOGY
C*****
      DIMENSION REAL(2),INTGR(2)
      DIMENSION ELQ(NDE)
      COMMON /BEGIN/ ICON,IKOUNT,ILNZ,IMASTR,IQ,IK
      COMMON /END/ LCON,LKOUNT,LLNZ,LMASTR,LQ,LK
C
      COMMON /DYNAM/ ISTART,I,J
C
C FIND STARTING LOCATION OF ELEMENT ASSEMBLY LIST, -1
      ISTART = IMASTR+LNUM-1
      ISTART = INTGR(ISTART)-1
C EXTRACT DISPLACEMENTS FROM Q AND STORE IN ELQ
      DO 1 I = 1,NDE
        J = ISTART+I
        J = IQ+INTGR(J)-1
        1 ELQ(I) = REAL(J)
      RETURN
      END

```

```

XTRA0001
XTRA0002
XTRA0003
XTRA0004
XTRA0005
XTRA0006
XTRA0007
XTRA0008
XTRA0009
XTRA0010
XTRA0011
XTRA0012
XTRA0013
XTRA0014
XTRA0015
XTRA0016
XTRA0017
XTRA0018
XTRA0019
XTRA0020
XTRA0021
XTRA0022
XTRA0023

```


APPENDIX E

Sample Output

DISTORTION ANALYSIS OF WELDED PANEL STRUCTURE....BY D. SHIN
 LX= 0.16000E 02, LY= 0.10000E 02, M=NO. ELEMENTS X DIR= 8
 NO. ELEMENTS Y DIR= 5, CX= 0.11300E 05, CY= 0.11300E 05
 THETA 0= 0.45000E-01 RADIANS, POISSONS RATIO= 0.30000E 00
 THICKNESS= 0.37500E 00, YOUNGS MODULUS= 0.30000E 08

NODE	W	$\frac{DW}{DX}$	$\frac{DW}{DY}$	$\frac{D(\frac{DW}{DY})}{DX}$
1	-0.91928E-01	0.0	0.0	0.0
2	-0.88355E-01	0.0	0.35772E-02	0.0
3	-0.77587E-01	0.0	0.72081E-02	0.0
4	-0.59449E-01	0.0	0.10954E-01	0.0
5	-0.33680E-01	0.0	0.14841E-01	0.0
6	0.0	0.0	0.18860E-01	0.0
7	-0.91103E-01	0.83277E-03	0.0	0.0
8	-0.87579E-01	0.78365E-03	0.35294E-02	-0.53047E-04
9	-0.76942E-01	0.65235E-03	0.71237E-02	-0.90185E-04
10	-0.58998E-01	0.45744E-03	0.10345E-01	-0.11218E-03
11	-0.33455E-01	0.22819E-03	0.14724E-01	-0.11746E-03
12	0.0	0.0	0.18753E-01	-0.10866E-03
13	-0.88539E-01	0.17606E-02	0.0	0.0
14	-0.85157E-01	0.16631E-02	0.33897E-02	-0.10264E-03
15	-0.74923E-01	0.13905E-02	0.68673E-02	-0.18373E-03
16	-0.57579E-01	0.97915E-03	0.10509E-01	-0.23550E-03
17	-0.32746E-01	0.49051E-03	0.14360E-01	-0.25204E-03
18	0.0	0.0	0.18416E-01	-0.23381E-03
19	-0.83915E-01	0.29142E-02	0.0	0.0
20	-0.80781E-01	0.27593E-02	0.31448E-02	-0.15960E-03
21	-0.71257E-01	0.23174E-02	0.64100E-02	-0.29382E-03
22	-0.54990E-01	0.16415E-02	0.99026E-02	-0.38679E-03
23	-0.31445E-01	0.82650E-03	0.13695E-01	-0.42260E-03
24	0.0	0.0	0.17798E-01	-0.39422E-03
25	-0.76646E-01	0.44285E-02	0.0	0.0
26	-0.73887E-01	0.42048E-02	0.27736E-02	-0.22747E-03
27	-0.65447E-01	0.35564E-02	0.57058E-02	-0.42997E-03
28	-0.50856E-01	0.25432E-02	0.89496E-02	-0.58395E-03
29	-0.29355E-01	0.12916E-02	0.12632E-01	-0.55582E-03
30	0.0	0.0	0.16801E-01	-0.61803E-03
31	-0.65869E-01	0.64462E-02	0.0	0.0
32	-0.63630E-01	0.61465E-02	0.22550E-02	-0.30303E-03
33	-0.56724E-01	0.52594E-02	0.46998E-02	-0.59186E-03
34	-0.44567E-01	0.38252E-02	0.75426E-02	-0.84170E-03
35	-0.26136E-01	0.19748E-02	0.11009E-01	-0.99117E-03

36	0.0	0.0	0.15257E-01	-0.95434E-03
37	-0.50438E-01	0.91026E-02	0.0	0.0
38	-0.48869E-01	0.87341E-02	0.15947E-02	-0.37215E-03
39	-0.43977E-01	0.76144E-02	0.33565E-02	-0.75791E-03
40	-0.35158E-01	0.57086E-02	0.55600E-02	-0.11564E-02
41	-0.21198E-01	0.30509E-02	0.85710E-02	-0.14943E-02
42	0.0	0.0	0.12358E-01	-0.15218E-02
43	-0.28973E-01	0.12489E-01	0.0	0.0
44	-0.28182E-01	0.12086E-01	0.80030E-03	-0.40724E-03
45	-0.25692E-01	0.10828E-01	0.17212E-02	-0.86705E-03
46	-0.21094E-01	0.85499E-02	0.29518E-02	-0.14512E-02
47	-0.13388E-01	0.49515E-02	0.49353E-02	-0.22364E-02
48	0.0	0.0	0.88895E-02	-0.27871E-02
49	0.0	0.16597E-01	0.0	0.0
50	0.0	0.16219E-01	0.0	-0.38120E-03
51	0.0	0.15027E-01	0.0	-0.82971E-03
52	0.0	0.12794E-01	0.0	-0.14703E-02
53	0.0	0.88826E-02	0.0	-0.27754E-02
54	0.0	0.0	0.0	-0.85932E-02

DISTORTION ANALYSIS OF WELDED PANEL STRUCTURE....BY D. SHIN
 LX= 0.16000E 02, LY= 0.10000E 02, M=NO. ELEMENTS X DIR= 8
 NO. ELEMENTS Y DIR= 5, CX= 0.42000E 05, CY= 0.42000E 05
 THETA 0= 0.55000E-01 RADIANS, POISSONS RATIO= 0.30000E 00
 THICKNESS= 0.37500E 00, YOUNGS MODULUS= 0.30000E 08

NODE	W	$\frac{DW}{DX}$	$\frac{DW}{DY}$	$\frac{D(\frac{DW}{DY})}{DX}$
1	-0.20630E 00	0.0	0.0	0.0
2	-0.19811E 00	0.0	0.81999E-02	0.0
3	-0.17348E 00	0.0	0.16446E-01	0.0
4	-0.13228E 00	0.0	0.24772E-01	0.0
5	-0.74396E-01	0.0	0.33102E-01	0.0
6	0.0	0.0	0.41245E-01	0.0
7	-0.20479E 00	0.15373E-02	0.0	0.0
8	-0.19671E 00	0.14253E-02	0.80926E-02	-0.12003E-03
9	-0.17237E 00	0.11358E-02	0.16265E-01	-0.19459E-03
10	-0.13156E 00	0.73612E-03	0.24561E-01	-0.21924E-03
11	-0.74079E-01	0.32457E-03	0.32912E-01	-0.19216E-03
12	0.0	0.0	0.41119E-01	-0.13083E-03
13	-0.19997E 00	0.33650E-02	0.0	0.0
14	-0.19222E 00	0.31367E-02	0.77720E-02	-0.23848E-03
15	-0.16378E 00	0.25173E-02	0.15705E-01	-0.40788E-03
16	-0.12922E 00	0.16449E-02	0.23897E-01	-0.47647E-03
17	-0.73044E-01	0.73174E-03	0.32306E-01	-0.43069E-03
18	0.0	0.0	0.40708E-01	-0.29460E-03
19	-0.19090E 00	0.58546E-02	0.0	0.0
20	-0.18374E 00	0.54795E-02	0.71915E-02	-0.38374E-03
21	-0.16195E 00	0.44364E-02	0.14669E-01	-0.67925E-03
22	-0.12473E 00	0.29342E-02	0.22639E-01	-0.82320E-03
23	-0.71033E-01	0.13205E-02	0.31135E-01	-0.76912E-03
24	0.0	0.0	0.39906E-01	-0.53212E-03
25	-0.17587E 00	0.93857E-02	0.0	0.0
26	-0.16963E 00	0.88291E-02	0.62826E-02	-0.56339E-03
27	-0.15045E 00	0.72449E-02	0.13005E-01	-0.10340E-02
28	-0.11705E 00	0.48832E-02	0.20552E-01	-0.13141E-02
29	-0.67547E-01	0.22379E-02	0.29125E-01	-0.12862E-02
30	0.0	0.0	0.38504E-01	-0.90881E-03
31	-0.15242E 00	0.14326E-01	0.0	0.0
32	-0.14748E 00	0.13576E-01	0.49887E-02	-0.75691E-03
33	-0.13209E 00	0.11375E-01	0.10551E-01	-0.14575E-02
34	-0.10449E 00	0.79106E-02	0.17302E-01	-0.19902E-02
35	-0.61697E-01	0.37420E-02	0.25812E-01	-0.21083E-02

36	0.0	0.0	0.36123E-01	-0.15539E-02
37	-0.11745E 00	0.20934E-01	0.0	0.0
38	-0.11416E 00	0.20047E-01	0.33369E-02	-0.89815E-03
39	-0.10373E 00	0.17337E-01	0.72541E-02	-0.18427E-02
40	-0.84243E-01	0.12707E-01	0.12542E-01	-0.28108E-02
41	-0.51826E-01	0.63954E-02	0.20411E-01	-0.34666E-02
42	0.0	0.0	0.31996E-01	-0.28367E-02
43	-0.67591E-01	0.29178E-01	0.0	0.0
44	-0.66083E-01	0.28315E-01	0.15351E-02	-0.87708E-03
45	-0.61216E-01	0.25567E-01	0.34282E-02	-0.19292E-02
46	-0.51732E-01	0.20381E-01	0.62956E-02	-0.34149E-02
47	-0.34341E-01	0.11729E-01	0.11718E-01	-0.56074E-02
48	0.0	0.0	0.24053E-01	-0.64475E-02
49	0.0	0.38519E-01	0.0	0.0
50	0.0	0.37908E-01	0.0	-0.62151E-03
51	0.0	0.35921E-01	0.0	-0.14222E-02
52	0.0	0.31951E-01	0.0	-0.27916E-02
53	0.0	0.24050E-01	0.0	-0.64398E-02
54	0.0	0.0	0.0	-0.28044E-01

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